551–657

preteach some of the key
is chapter. Particularly for
e Learners (ELL),
ocabulary before the
n begins gives students a
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s a useful technique.

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CHAPTER 11

What You'll Learn in Chapter 11

- How to find square roots
- How to simplify radical expressions
- How to use the Pythagorean theorem to find missing lengths of a right triangle
- How to solve radical equations

Skills & Concepts You Need for Chapter 11

1-2 Simplify.

1. $\frac{18}{50}$ **2.** $\frac{18}{66}$ **3.** $\frac{81}{27}$ **4.** $\frac{100}{50}$

1-3 What is the meaning of each?

5. 5^2 **6.** 4^3 **7.** x^5 **8.** 7^6

2-1 Simplify.

9. $|-8|$ **10.** $|-15|$ **11.** $|0|$

2-5 Multiply.

12. $\frac{5}{3} \cdot \frac{5}{3}$ **13.** $\left(-\frac{2}{9}\right) \cdot \frac{2}{9}$ **14.** $\left(-\frac{3}{16}\right) \cdot \left(-\frac{3}{16}\right)$

15. $\frac{11}{4} \cdot \left(-\frac{11}{4}\right)$ **16.** $(-5)(-5)$ **17.** $(-6)(-6)(-6)$

18. $\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)$ **19.** $\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)$

5-1 Multiply or divide. Simplify.

20. $x^3 \cdot x^3$ **21.** $(a^2b^3)(ab^2)$

22. $\frac{x^9}{x^4}$ **23.** $\frac{ab^3}{ab}$

6-1 to 6-5 Factor.

24. $x^3 - x^2$ **25.** $5x - 30x^2$

26. $x^2 + 2x + 1$ **27.** $x^2 - 14x + 49$

Skills & Concepts You Need for Chapter 11

1. $\frac{9}{25}$
2. $\frac{3}{11}$
3. 3
4. 2
5. 5×5
6. $4 \times 4 \times 4$
7. $x \times x \times x \times x \times x$
8. $7 \times 7 \times 7 \times 7 \times 7 \times 7$
9. 8
10. 15
11. 0
12. $\frac{25}{9}$
13. $-\frac{4}{81}$
14. $\frac{9}{256}$
15. $-\frac{121}{16}$
16. 25
17. -216
18. $\frac{9}{16}$
19. $-\frac{1}{125}$
20. x^6

22. a^3b^5
22. x^5
23. b^2
24. $x^2(x - 1)$
25. $5x(1 - 6x)$
26. $(x + 1)^2$
27. $(x - 7)^2$

›roximating Irrational nbers

›oint

ı, the mathematician Johann :h Lambert presented to the Academy the first proof that ational. The decimal entation of π never ends and epeats. It is possible to ‹imate π to any degree of cy by a fraction. Here are ational approximations to π. :cimals are given far enough w the deviation of each ‹imation from the true value

41592653589793238462643

= 3.0

= 3.1428

= 3.141509

= 3.1415929

= 3.141592653

ıstions

›etween what two integers?

between what two integers?

is between what two integers? 13

ıard Example

calculator or Table 1 to ›ximate $\sqrt{13}$.

$\approx$ 3.606, rounded to 3 decimal

out that a calculator cannot an irrational number. A lator stores a rational ›ximation in the form of a inating decimal.

ɔu may want students to try to a perfect square root of 2. ents may conclude that 2136 is a perfect square root Check this by reentering the ›er and squaring.

The square roots of most whole numbers are irrational. Only the perfect squares 0, 1, 4, 9, 16, 25, 36, and so on have rational square roots.

EXAMPLES Identify the rational numbers and the irrational numbers.

4 $\sqrt{3}$ $\sqrt{3}$ is irrational, since 3 is not a perfect square.

5 $\sqrt{25}$ $\sqrt{25}$ is rational, since 25 is a perfect square.

6 $\sqrt{35}$ $\sqrt{35}$ is irrational, since 35 is not a perfect square.

7 $-\sqrt{49}$ $-\sqrt{49}$ is rational, since 49 is a perfect square.

Try This Identify the rational numbers and the irrational numbers.

d. $\sqrt{5}$ **e.** $-\sqrt{36}$ **f.** $-\sqrt{32}$ **g.** $\sqrt{101}$

PART 3 Approximating Irrational Numbers

Objective: Use a table or a calculator to give an approximation for an irrational number.

We can use a rational number to approximate an irrational number. Table 1 in the Appendix contains rational approximations for square roots. We can also use a calculator to find rational approximations for square roots.

EXAMPLE 8 Approximate $\sqrt{10}$.

Using Table 1, find 10 in the first column headed N. Look in the third column headed $\sqrt{N}$ opposite 10. Thus $\sqrt{10} \approx 3.162$. The symbol $\approx$ means "is approximately equal to."

Calculators are very useful for finding square roots.

Square Roots

Most calculators have a square root key that is usually accessed using an INV or 2nd key with the x^2 key.

$\sqrt{10} \rightarrow 10$ 2nd x^2 $\rightarrow$ 3.1622777

Since $\sqrt{10}$ is an irrational number, the decimal shown on the calculator is a rational approximation.

Using either method, we find $\sqrt{10} \approx 3.162$ to the nearest thousandth.

Try This Approximate each square root to the nearest thousandth.

h. $\sqrt{7}$ **i.** $\sqrt{72}$ **j.** $\sqrt{18}$ **k.** $\sqrt{45}$

Try This

d. Irrational
e. Rational
f. Irrational
g. Irrational
h. 2.646
i. 8.485
j. 4.243
k. 6.708

Exercises

1. 1, −1
2. 2, −2
3. 4, −4
4. 11, −11
5. 13, −13
6. 18, −18
7. 2
8. −3
9. −5
10. −8
11. −9
12. −15
13. 20
14. 19

11-1 Exercises

A

Find the square roots of each number.

1. 1 2. 4 3. 16 4. 121 5. 169 6. 324

Simplify.

7. $\sqrt{4}$ 8. $-\sqrt{9}$ 9. $-\sqrt{25}$ 10. $-\sqrt{64}$
11. $-\sqrt{81}$ 12. $-\sqrt{225}$ 13. $\sqrt{400}$ 14. $\sqrt{361}$
15. $\sqrt{196}$ 16. $\sqrt{289}$ 17. $-\sqrt{36}$ 18. $\sqrt{625}$

Identify each square root as rational or irrational.

19. $\sqrt{8}$ 20. $\sqrt{49}$ 21. $\sqrt{100}$ 22. $\sqrt{75}$
23. $-\sqrt{4}$ 24. $-\sqrt{12}$ 25. $-\sqrt{125}$ 26. $-\sqrt{196}$

Approximate these square roots to the nearest thousandth.

27. $\sqrt{5}$ 28. $\sqrt{17}$ 29. $\sqrt{93}$ 30. $\sqrt{40}$ 31. $\sqrt{54}$ 32. $\sqrt{111}$

B

Identify each number as rational or irrational. If it is rational, is it best described as a whole number, an integer, or a rational number?

33. $\sqrt{120}$ 34. $\sqrt{0.49}$ 35. $\sqrt{196}$ 36. $-\sqrt{215}$

Simplify.

37. $\sqrt{\sqrt{16}}$ 38. $\sqrt{3^2 + 4^2}$ 39. $\sqrt{(3 + 4)^2}$ 40. $(\sqrt{5} + 13)^2$

41. **TEST PREP** Which expression has the least value?
  A. $\sqrt{(5 + 6)^2}$ B. $\sqrt{5^2 + 6^2}$ C. $\left(\sqrt{5 + 6}\right)^2$
  D. $\sqrt{5^2} + \sqrt{6^2}$ E. The expressions have the same value.
42. Between what two consecutive integers is $-\sqrt{33}$?
43. Between what two consecutive integers is $-\sqrt{57}$?

Determine whether each statement is sometimes, always, or never true.

44. The expression $\sqrt{a^2 + b^2}$ is irrational.
45. The expression $\sqrt{(a + b)^2}$ is irrational.
46. If $\sqrt{a}$ is an integer, then $-\sqrt{a}$ is also an integer.

Challenge

47. 10.63 could be an approximation for the square root of what integer?
48. What number is halfway between x and y?
49. ***Critical Thinking*** Find a number that is the square of an integer and the cube of a different integer.
50. A formula for the energy of an object of mass (m) and velocity (v) is given by $E = \frac{1}{2}mv^2$. Find v in meters/second to the nearest tenth if $E = 20$ joules, and $m = 8$ kilograms.

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

3. PRACTICE/ASSESS

LESSON QUIZ

1. Find the square roots of 256.
   $\pm\sqrt{256} = 16$ or -16
2. Simplify $\sqrt{900}$.
   $\sqrt{900} = 30$
3. Is $\sqrt{75}$ rational or irrational?
   75 is not a perfect square, henc
   is irrational.
4. Use Table 1 or a calculator to fi
   to three decimal places.
   $\sqrt{17} = 4.123$

Assignment Guide

To provide flexible scheduling,
lesson can be split into parts.

1 Core 1–18, 37–43
  Extension 48–51
2 Core 19–26, 33–36
  Extension 44–46
3 Core 27–32
  Extension 47

Use Mixed Review to maintain s

Exercises

15. 14
16. 17
17. −6
18. 25
19. Irrational
20. Rational
21. Rational
22. Irrational
23. Rational
24. Irrational
25. Irrational
26. Rational
27. 2.236
28. 4.123
29. 9.644
30. 6.325
31. 7.348
32. 10.536
33. Irrational
34. Rational, rational
35. Rational, whole number
36. Irrational
37. 2
38. 5
39. 7
40. 18
41. B
42. −5, −6
43. −8, −7
44. Sometimes
45. Sometimes
46. Always
47. 113
48. $\frac{x + y}{2}$
49. Possible answers:
    1 is $(-1)^2$ and 1^3, 64 is 8^2 and 4^3
50. $\sqrt{5} \approx 2.2$ meters/second

itional Number?

of by contradiction can be
y prime number.

Exercises
51. a. 3
b. 43
c. −6
d. 94
Mixed Review
52. $\frac{x^4 y}{-3}$
53. $\frac{x+1}{4}$
54. $\frac{x^2(x+1)}{15}$
55. $x+2$
56. $\frac{-2x}{(x+2)(x-2)}$

51. Find y if $\sqrt{y+6}$ is
a. 3 **b.** 7 **c.** 0 **d.** 10

Mixed Review

Simplify. **52.** $\frac{x^5y^2}{-3xy}$ **53.** $\frac{5(x+1)}{8x} \cdot \frac{4x}{10}$ **54.** $\frac{3(x+1)}{5x^2} \cdot \frac{x^4}{9}$

55. $\frac{x^2+8x}{x+3} - \frac{3x+2}{x+3} + \frac{8}{x+3}$ **56.** $\frac{3x-10}{x^2-4} - \frac{5}{x+2}$ *10-1, 10-2, 10-4, 10-5*

Find the LCM. **57.** $25x - 10, 5x + 2$ **58.** $a + c, a - c$ *10-5*

Solve. **59.** $\frac{2}{3} + \frac{1}{4} = \frac{x}{6}$ **60.** $\frac{5}{8} + \frac{1}{2} = \frac{y}{24}$ **61.** $\frac{4}{9} - \frac{1}{3} = \frac{x}{36}$

62. $\frac{2}{3} - \frac{1}{5} = \frac{1}{a}$ **63.** $\frac{1}{6} + \frac{4}{9} = \frac{1}{a}$ **64.** $\frac{1}{4} - \frac{5}{6} = \frac{1}{b}$ *10-6*

Divide. **65.** $(x^2 - 9x + 20) \div (x - 4)$ **66.** $(y^4 - 81) \div (y - 3)$ *10-9*

Solve. **67.** $y - 6x = 1$, $5x + y = 12$ **68.** $4x + y = -4$, $10x + 3y = -9$ *8-1, 8-2, 8-3*

California Topic

CA 25.1: Use properties of numbers to construct indirect arguments.

Is $\sqrt{7}$ a Rational Number?

We can show that $\sqrt{7}$ is not a rational number by using an indirect proof, or *proof by contradiction*. We will use a fact proved in more advanced courses. If p is a prime factor of a^2, then p is a factor of a.

First, we assume that $\sqrt{7}$ is a rational number, which we write in simplest form as $\frac{a}{b}$ (where a and b are integers and $b \neq 0$). Since $\frac{a}{b}$ is in simplest form, we know that a and b have no common factors greater than 1.

$\sqrt{7} = \frac{a}{b}$ Assuming the opposite of what we wish to prove

$7 = \frac{a^2}{b^2}$ Squaring both sides

$7b^2 = a^2$ Multiplying both sides by b^2

The last equation shows that 7 is a factor of a^2. Since 7 is a prime number, 7 must also be a factor of a. So $a = 7k$ for some integer k.

$7b^2 = (7k)^2$ Substituting $7k$ for a

$7b^2 = 49k^2$ Raising a product to a power

$b^2 = 7k^2$ Multiplying both sides by $\frac{1}{7}$

The last equation shows that 7 is a factor of b^2, and hence of b. Thus a and b have 7 as a common factor. But a and b have no common factor greater than 1. This contradiction shows our assumption that $\sqrt{7}$ is rational to be false. Therefore, $\sqrt{7}$ is an irrational number.

Exercise Show that $\sqrt{5}$ is irrational.

Writing Math

In an **indirect proof**, we assume that the opposite of what we wish to prove is true. Then we show that this assumption leads to a contradiction. Since it leads to a contradiction our assumption must be false, and what we wish to prove must be true.

57. $5(5x+2)(5x-2)$
58. $(a+c)(a-c)$
59. $\frac{11}{2}$
60. 27
61. 4
62. $\frac{15}{7}$
63. $\frac{18}{11}$
64. $-\frac{12}{7}$
65. $x - 5$
66. $y^3 + 3y^2 + 9y + 27$
67. (1, 7)
68. $(-\frac{3}{2}, 2)$

Is $\sqrt{7}$ a Rational Number?

Exercise

Assume that $\sqrt{5}$ is a rational number, written $\frac{a}{b}$ in simplest form.

$\sqrt{5} = \frac{a}{b}$

$5 = \frac{a^2}{b^2}$

$5b^2 = a^2$

This shows that 5 is a factor of a^2. Since 5 is a prime number, 5 must also be a factor of a.

$5b^2 = (5k)^2$

$5b^2 = 25k^2$

$b^2 = 5k^2$

This shows that 5 is a factor of b^2, and hence of b. Thus a and b have 5 as a common factor. But a and b have no common factor greater than 1. Therefore the assumption is false, and $\sqrt{5}$ is an irrational number.

view

1)

2x
)(x − 2)

11-2 Exercises

Extra Help On the Web
Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A

1. Evaluate the expression $\sqrt{3x - 12}$ for $x = 4$. Is the result a real number?
2. Evaluate the expression $\sqrt{8 - 4y}$ for $y = 10$. Is the result a real number?
3. Evaluate $\sqrt{x + 12}$ for $x = -6$. Is the result a real number?
4. Evaluate $\sqrt{3y + 12}$ for $y = -5$. Is the result a real number?

Determine the values of the variable that make each expression a real number.

5. $\sqrt{5x}$ 6. $\sqrt{3y}$ 7. $\sqrt{t - 5}$ 8. $\sqrt{y - 8}$
9. $\sqrt{y + 8}$ 10. $\sqrt{x + 6}$ 11. $\sqrt{x + 20}$ 12. $\sqrt{m - 18}$
13. $\sqrt{2y - 7}$ 14. $\sqrt{3x + 8}$ 15. $\sqrt{t^2 + 5}$ 16. $\sqrt{y^2 + 1}$

Simplify.

17. $\sqrt{t^2}$ 18. $\sqrt{x^2}$ 19. $\sqrt{9x^2}$ 20. $\sqrt{4a^2}$
21. $\sqrt{(-7)^2}$ 22. $\sqrt{(-5)^2}$ 23. $\sqrt{(-4d)^2}$ 24. $\sqrt{(-3b)^2}$
25. $\sqrt{(x + 3)^2}$ 26. $\sqrt{(x - 7)^2}$ 27. $\sqrt{a^2 - 10a + 25}$
28. $\sqrt{x^2 + 2x + 1}$ 29. $\sqrt{4a^2 - 4a + 1}$ 30. $\sqrt{9a^2 - 12a + 4}$

B

Solve.

31. $\sqrt{x^2} = 6$ 32. $\sqrt{y^2} = -7$ 33. $-\sqrt{x^2} = -3$
34. $t^2 = 49$ 35. $\sqrt{(x - 3)^2} = 5$ 36. $\sqrt{4a^2 - 12a + 9} = 3$

Simplify.

37. $\sqrt{(3a)^2}$ 38. $\sqrt{(4a)^2(4a)^2}$ 39. $\sqrt{\frac{144x^8}{36y^6}}$
40. $\sqrt{\frac{y^{12}}{8100}}$ 41. $\sqrt{\frac{169}{m^{16}}}$ 42. $\sqrt{\frac{p^2}{3600}}$

43. Determine the values for the variable that will make each expression a real number.
    a. $\sqrt{m(m + 3)}$ b. $\sqrt{x^2(x - 3)}$
44. ***Critical Thinking*** Given a and c, what must be true of b to make $\sqrt{b^2 - 4ac}$ a real number?
    a. $a = -3, c = 2$ b. $a = 2, c = 8$

Determine whether each statement is sometimes, always, or never true.

45. $\sqrt{a^2 + b^2}$ is a real number. 46. $\sqrt{3 - t}$ is a real number for $t \geq 3$.
47. $\sqrt{a^2 - b^2}$ is a real number. 48. $\sqrt{a^2 + ab + b^2}$ is a real number.

3. PRACTICE/ASSESS

LESSON QUIZ

1. For what values of x is $\sqrt{x^2 -}$
   $x \leq -2$ or $x \geq 2$
2. Simplify $\sqrt{16x^2}$.
   $4|x|$
3. Simplify $\sqrt{x^2 + 8x + 16}$.
   $|x + 4|$

Assignment Guide

To provide flexible scheduling, lesson can be split into parts.
1 Core 1–16, 43–48
  Extension 49–53
2 Core 17–30, 31–38
  Extension 39–42

Use Mixed Review to maintain

Exercises

1. 0; Yes
2. $\sqrt{-32}$; No
3. $\sqrt{6}$; Yes
4. $\sqrt{-3}$; No
5. $x \geq 0$
6. $y \geq 0$
7. $t \geq 5$
8. $y \geq 8$
9. $y \geq -8$
10. $x \geq -6$
11. $x \geq -20$
12. $m \geq 18$
13. $y \geq \frac{7}{2}$
14. $x \geq -\frac{8}{3}$
15. Any value
16. Any value
17. $|t|$
18. $|x|$
19. $3|x|$
20. $2|a|$
21. 7
22. 5
23. $4|d|$
24. $3|b|$
25. $|x + 3|$
26. $|x - 7|$
27. $|a - 5|$
28. $|x + 1|$
29. $|2a - 1|$
30. $|3a - 2|$
31. 6, −6
32. No value
33. 3, −3
34. 7, −7
35. −2, 8
36. 0, 3
37. $3|a|$
38. $16a^2$
39. $\frac{2x^4}{|y^3|}$
40. $\frac{y^6}{90}$
41. $\frac{13}{m^8}$
42. $\frac{|p|}{60}$

43a. $m \geq 0$ or $m \leq -3$
 b. $x \geq 3$ or $x = 0$
44a. Any real number
 b. $b \leq -8$ or $b \geq 8$
45. Always
46. Sometimes
47. Sometimes
48. Always

Challenge

Determine the values for the variable that will make each expression a real number.

49. $\sqrt{(x+3)(x-2)}$ **50.** $\sqrt{x^2+7x+12}$

51. $\sqrt{x^2-4}$ **52.** $\sqrt{4x^2-1}$

53. For a polynomial of the form $ax^2+bx+c=0$ to have real solutions, $\sqrt{b^2-4ac}$ must be a real number. Which of the following polynomials have real solutions?

a. $x^2-12x+3=0$ **b.** $x^2+2x-50=0$

c. $x^2+5x+7=0$ **d.** $5x^2+2x+1=0$

e. $-x^2+x+1=0$ **f.** $-x^2+x-1=0$

Mixed Review

Solve. **54.** $|x+3|<1$ **55.** $|x-9|=-3$ **56.** $|y+3|\geq 5$ *9-3, 9-4*

Simplify. **57.** $\frac{x-3}{x+2}-\frac{3x+1}{x+2}$ **58.** $\frac{6x+7}{x-3}-\frac{2x+3}{x-3}$

59. $\frac{5}{y+5}+\frac{2}{(y+5)^2}$ **60.** $\frac{5}{y+4}-\frac{3}{y+3}$ *10-4, 10-5*

Solve. **61.** $x-\frac{6}{x}=5$ **62.** $\frac{6}{x}=\frac{5}{x}+\frac{1}{2}$ **63.** $\frac{y+1}{y-3}=2$ *10-6*

The price of the larger can is \$2.99. The price of the smaller can is \$.89. Refer to the volumes in Exercise 3 to find the price per cubic centimeter. Which can is the better buy?

Connections: Geometry

The formula for the volume of a cylinder is $V=\pi r^2h$. The height of each cylinder and the volume are given. Find the radius of each figure. If you do not have a calculator with a $\boxed{\pi}$ key, use 3.14 for π. Round the radius to the nearest whole number.

1.

$h = 25$ in., $V = 2826$ in.^{3}

2.

$h = 6.2$ cm, $V = 175$ cm^3

3. Use the photo at the left. The height of the larger can is 17 cm, and the volume is 3208 cm^3. The height of the smaller can is 11 cm, and the volume is 486 cm^3. Find the radius of each can.

Exercises

49. $x \geq 2$ or $x \leq -3$
50. $x \leq -4$ or $x \geq -3$
51. $x \geq 2$ or $x \leq -2$
52. $x \geq \frac{1}{2}$ or $x \leq -\frac{1}{2}$
53. a. Real
 b. Real
 c. Not real
 d. Not real
 e. Real
 f. Not real

Mixed Review

54. $-4 < x < -2$
55. No solution
56. $y \leq -8$ or $y \geq 2$
57. -2
58. $\frac{4(x+1)}{x-3}$
59. $\frac{5y+27}{(y+5)^2}$
60. $\frac{2y+3}{(y+3)(y+4)}$
61. 6, -1
62. 2
63. 7

Connections: Geometry

1. 6 in.
2. 3 cm
3. larger can, 7.75 cm
 smaller can, 3.75 cm

Photo Caption

larger can, 0.09¢/cm^3
smaller can, 0.18¢/cm^3
The larger can is a better buy.

ırd Examples

$= \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

$\sqrt{x} = 7\sqrt{x}$

$\overline{3t} = 2\sqrt{13t}$

$+ 20x + 50$

$+ 10x + 25) = \sqrt{2}(x + 5)$

$\overline{2} = x^6$

$)^2 = x^{13}$

$= \sqrt{(x^6)^2 x} = x^6\sqrt{x}$

EXAMPLES Simplify.

2 $\sqrt{48t} = \sqrt{16 \cdot 3t}$ — Identifying perfect square factors

$= \sqrt{16} \cdot \sqrt{3t}$ — Using the product property for radicals

$= 4\sqrt{3t}$

3 $\sqrt{72x^2} = \sqrt{36x^2 \cdot 2}$ — Identifying perfect square factors

$= \sqrt{36x^2} \cdot \sqrt{2}$ — Using the product property for radicals

$= 6x\sqrt{2}$

4 $\sqrt{3x^2 + 6x + 3} = \sqrt{3(x^2 + 2x + 1)}$

$= \sqrt{3} \cdot \sqrt{x^2 + 2x + 1}$ — Using the product property for radicals

$= \sqrt{3} \cdot \sqrt{(x + 1)^2}$

$= \sqrt{3}(x + 1)$

Try This Factor and simplify.

a. $\sqrt{32}$ **b.** $\sqrt{25x^2}$ **c.** $\sqrt{60x}$

d. $\sqrt{45x^2}$ **e.** $\sqrt{7x^2 - 14x + 7}$

To take a square root of a power such as x^8, the exponent must be even. We then take half the exponent. Recall that $(x^4)^2 = x^8$.

EXAMPLES Simplify.

5 $\sqrt{x^6} = \sqrt{(x^3)^2}$

$= x^3$

6 $\sqrt{x^{22}} = x^{11}$

When odd powers occur, express the power as the product of the largest even power and x. Then simplify the even power.

EXAMPLE 7 Simplify.

$\sqrt{x^9} = \sqrt{x^8 \cdot x}$

$= \sqrt{x^8} \cdot \sqrt{x}$

$= x^4\sqrt{x}$

Try This Simplify.

f. $\sqrt{y^8}$ **g.** $\sqrt{(x + y)^{14}}$ **h.** $\sqrt{t^{15}}$ **i.** $\sqrt{a^{25}}$

Try This

a. $4\sqrt{2}$
b. $5x$
c. $2\sqrt{15x}$
d. $3x\sqrt{5}$
e. $\sqrt{7}(x - 1)$
f. y^4
g. $(x + y)^7$
h. $t^7\sqrt{t}$
i. $a^{12}\sqrt{a}$

Exercises

1. $2\sqrt{3}$
2. $2\sqrt{2}$
3. $2\sqrt{5}$
4. $3\sqrt{5}$
5. $5\sqrt{3}$
6. $5\sqrt{2}$
7. $10\sqrt{2}$
8. $10\sqrt{3}$
9. $x\sqrt{3}$
10. $y\sqrt{5}$
11. $4\sqrt{a}$
12. $7\sqrt{b}$
13. $x\sqrt{13}$
14. $t\sqrt{29}$
15. $3\sqrt{x}$
16. $2\sqrt{y}$
17. $8y$
18. $3x$
19. $2t\sqrt{2}$
20. $5a\sqrt{5}$
21. $2(x + 1)$
22. $(x + 2)\sqrt{3}$
23. $(x + 3)\sqrt{2}$
24. $(x + 3)\sqrt{5}$
25. $2x + 3y$
26. $(x + 5y)\sqrt{3}$
27. x^3
28. x^5
29. x^6
30. x^8
31. $x^2\sqrt{x}$

11-3 Exercises

A

Simplify. Assume that all variables are nonnegative.

1. $\sqrt{12}$ 2. $\sqrt{8}$ 3. $\sqrt{20}$ 4. $\sqrt{45}$
5. $\sqrt{75}$ 6. $\sqrt{50}$ 7. $\sqrt{200}$ 8. $\sqrt{300}$
9. $\sqrt{3x^2}$ 10. $\sqrt{5y^2}$ 11. $\sqrt{16a}$ 12. $\sqrt{49b}$
13. $\sqrt{13x^2}$ 14. $\sqrt{29t^2}$ 15. $\sqrt{9x}$ 16. $\sqrt{4y}$
17. $\sqrt{64y^2}$ 18. $\sqrt{9x^2}$ 19. $\sqrt{8t^2}$ 20. $\sqrt{125a^2}$
21. $\sqrt{4x^2 + 8x + 4}$ 22. $\sqrt{3x^2 + 12x + 12}$
23. $\sqrt{2x^2 + 12x + 18}$ 24. $\sqrt{5x^2 + 30x + 45}$
25. $\sqrt{4x^2 + 12xy + 9y^2}$ 26. $\sqrt{3x^2 + 30xy + 75y^2}$

Simplify.

27. $\sqrt{x^6}$ 28. $\sqrt{x^{10}}$ 29. $\sqrt{x^{12}}$
30. $\sqrt{x^{16}}$ 31. $\sqrt{x^5}$ 32. $\sqrt{x^3}$
33. $\sqrt{t^{19}}$ 34. $\sqrt{p^{17}}$ 35. $\sqrt{(y-2)^8}$
36. $\sqrt{(x+3)^6}$ 37. $\sqrt{4(x+5)^{10}}$ 38. $\sqrt{16(a-7)^4}$
39. $\sqrt{36m^3}$ 40. $\sqrt{250y^3}$ 41. $\sqrt{8a^5}$
42. $\sqrt{12b^7}$ 43. $\sqrt{448x^6y^3}$ 44. $\sqrt{243x^5y^4}$

45. ***Error Analysis*** What error did the student make in simplifying the radical expression?

$$\sqrt{9x^2y^3} = \sqrt{9x^2y^2}\sqrt{y} = 9xy\sqrt{y}$$

B

Simplify. Assume that all variables are nonnegative real numbers.

46. $3\sqrt{200}$ 47. $2\sqrt{75}$
48. $4\sqrt{12}$ 49. $-3\sqrt{72}$
50. $-2\sqrt{1000}$ 51. $6\sqrt{36x}$
52. $4m\sqrt{20m^2}$ 53. $2x\sqrt{50x^4}$
54. $5r^2\sqrt{32r^4s^3}$ 55. $3a^3\sqrt{28a^3b^5}$

Evaluate and simplify for $r = 5$ and $s = \sqrt{5}$.

56. $\sqrt{3 + r^2}$ 57. $\sqrt{r^2 - 1}$ 58. $\sqrt{r + s^2}$ 59. $\sqrt{50 - s^2}$

60. ***Critical Thinking*** Find $\sqrt{49}$, $\sqrt{490}$, $\sqrt{4900}$, $\sqrt{49{,}000}$, and $\sqrt{490{,}000}$. What pattern do you see?

Extra Help On the Web

Internet Connection

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

Practice Multiple Choice

California Algebra Standards

Choose the best answer.

1. Simplify $\sqrt{\frac{36x^{16}}{4900}}$.

A $-\frac{3x^4}{35}$

B $\frac{3x^4}{35}$

C $\frac{6}{7}x^4$

D $\frac{3x^8}{35}$

2. Which statement justifies information in the given equation?

$\sqrt{x^2 - 10x + 25} = |x - 5|$

F The radicand is never negative.

G The radicand is not a real number.

H Binomials cannot be squared to form trinomials.

J The absolute value is necessary to produce a positive value.

1. D; Algebra 2.0
2. J; Algebra 25.0

3. PRACTICE/ASSESS

LESSON QUIZ

Simplify.

1. $\sqrt{28}$
$\sqrt{4 \cdot 7} = 2\sqrt{7}$
2. $\sqrt{7x^2 + 14x + 7}$
$\sqrt{7(x^2 + 2x + 1)} = \sqrt{7}(x +$
3. $\sqrt{x^{14}}$
x^7
4. $\sqrt{9(a+2)^2}$
$3(a + 2)$

Assignment Guide

Core 1–59
Extension 60–70

Use Mixed Review to maintai

Exercises

32. $x\sqrt{x}$
33. $t^9\sqrt{t}$
34. $p^8\sqrt{p}$
35. $(y - 2)^4$
36. $(x + 3)^3$
37. $2(x + 5)^5$
38. $4(a - 7)^2$
39. $6m\sqrt{m}$
40. $5y\sqrt{10y}$
41. $2a^2\sqrt{2a}$
42. $2b^3\sqrt{3b}$
43. $8x^3y\sqrt{7y}$
44. $9x^2y^2\sqrt{3x}$
45. $\sqrt{9x^2y^2} = 3xy$, not $9xy$
46. $30\sqrt{2}$
47. $10\sqrt{3}$
48. $8\sqrt{3}$
49. $-18\sqrt{2}$
50. $-20\sqrt{10}$
51. $36\sqrt{x}$
52. $8m^2\sqrt{5}$
53. $10x^3\sqrt{2}$
54. $20r^4s\sqrt{2s}$
55. $6a^4b^2\sqrt{7ab}$
56. $2\sqrt{7}$
57. $2\sqrt{6}$
58. $\sqrt{10}$
59. $3\sqrt{5}$
60. $\sqrt{49} = 7$ $\sqrt{490} = 7\sqrt{10}$
$\sqrt{4900} = 70$ $\sqrt{49{,}000} = 70\sqrt{10}$
$\sqrt{490{,}000} = 700$
$\sqrt{49 \times 10^n} = \sqrt{49} \cdot \sqrt{10^n} = 7\sqrt{10^n}$

Roots

simplifying cube roots, it is helpful
nge the factors into two groups,
hat are perfect cubes and those
e not. You may want to have
ts practice finding perfect cube
f the following.
25, 216, 1000, a^{12}
6, 10, a^4

Challenge

Use the proper symbol (>, <, or =) between each pair of values. Assume
is positive.

61. $15 \square \sqrt{14}$ **62.** $15\sqrt{2} \square \sqrt{450}$ **63.** $16 \square \sqrt{15}\sqrt{17}$
64. $3\sqrt{11} \square 7\sqrt{2}$ **65.** $5\sqrt{7} \square 4\sqrt{11}$ **66.** $8 \square \sqrt{15}\sqrt{17}$
67. $3\sqrt{x} \square 2\sqrt{2.5x}$ **68.** $4\sqrt{x} \square 5\sqrt{0.64x}$
69. $90\sqrt{100x} \square 100\sqrt{90x}$ **70.** $4\sqrt{5x} \square \sqrt{12x} + 4\sqrt{2x}$

Mixed Review

Solve. **71.** $5x + y = -9$, $2x - y = 2$ **72.** $2x + 3y = 11$, $5y - x = 1$ **73.** $5y + 3x = -1$, $2x - 2y = 10$ *8-2, 8-3*

Simplify. **74.** $\frac{6a^2 + 24}{12a^2 + 18a + 42}$ **75.** $\frac{(x + 3)}{x^2} \cdot \frac{(x + 2)}{(x + 3)}$ *10-1, 10-2*

Solve. **76.** The sum of two numbers is 51 and the difference of the two numbers is 19. Find the two numbers. *8-6*

California Topic

CA 2.0: Understand and use the operation of taking a root.

Cube Roots

Objective: Find cube roots.

The number c is called the **cube root** of a if $c^3 = a$. We write this as $c = \sqrt[3]{a}$.

$$\sqrt[3]{216} = 6, \text{ since } 6^3 = 216 \text{ and } \sqrt[3]{-216} = -6, \text{ since } (-6)^3 = -216$$

The procedures you have learned for multiplying, factoring, and simplifying expressions involving square roots also apply to cube roots.

EXAMPLE Simplify.

$$\begin{aligned}\sqrt[3]{16a^4} &= \sqrt[3]{8 \cdot 2 \cdot a^3 \cdot a} && \text{Identifying factors that are perfect cubes}\\ &= \sqrt[3]{2^3 \cdot 2 \cdot a^3 \cdot a}\\ &= \sqrt[3]{2^3 \cdot a^3} \cdot \sqrt[3]{2a}\\ &= 2a\sqrt[3]{2a}\end{aligned}$$

Exercises

Simplify.

1. $\sqrt[3]{8}$ **2.** $\sqrt[3]{27}$ **3.** $\sqrt[3]{125}$ **4.** $\sqrt[3]{1000}$
5. $\sqrt[3]{-8}$ **6.** $\sqrt[3]{-64}$ **7.** $\sqrt[3]{-1}$ **8.** $\sqrt[3]{-8000}$
9. $\sqrt[3]{x^3}$ **10.** $\sqrt[3]{y^3}$ **11.** $\sqrt[3]{a^3b^3}$ **12.** $\sqrt[3]{mn^3}$
13. $\sqrt[3]{27x^6y^3}$ **14.** $\sqrt[3]{125m^9n^{12}}$ **15.** $\sqrt[3]{-8a^6b^9}$ **16.** $\sqrt[3]{-125p^{12}}$

Exercises

61. >
62. =
63. >
64. >
65. <
66. <
67. <
68. =
69. <
70. <

Mixed Review

71. (−1, −4)
72. (4, 1)
73. (3, −2)
74. $\frac{a^2 + 4}{2a^2 + 3a + 7}$
75. $\frac{x + 2}{x^2}$
76. 35, 16

CUBE ROOTS

Exercises

1. 2
2. 3
3. 5
4. 10
5. −2
6. −4
7. −1
8. −20
9. x
10. y
11. ab
12. $n\sqrt[3]{m}$
13. $3x^2y$
14. $5m^3n^4$
15. $-2a^2b^3$
16. $-5p^4$

ACTICE/ASSESS

l QUIZ

:iply $\sqrt{7} \cdot \sqrt{11}$.

:iply $\sqrt{x+1} \cdot \sqrt{x-1}$.
$\overline{+ 1)(x - 1)} = \sqrt{x^2 - 1}$

:iply and simplify $\sqrt{2} \cdot \sqrt{50}$.

:iply and simplify $\sqrt{3x} \cdot \sqrt{12x}$.

gnment Guide
ore 1–54
xtension 55–57

Mixed Review to maintain skills.

Extra Help On the Web
Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

11-4 Exercises

A

Multiply.

1. $\sqrt{2} \cdot \sqrt{3}$ **2.** $\sqrt{3} \cdot \sqrt{5}$ **3.** $\sqrt{17} \cdot \sqrt{17}$
4. $\sqrt{25} \cdot \sqrt{3}$ **5.** $\sqrt{2} \cdot \sqrt{x}$ **6.** $\sqrt{x} \cdot \sqrt{x-3}$
7. $\sqrt{5} \cdot \sqrt{2x-1}$ **8.** $\sqrt{x+2} \cdot \sqrt{x+1}$ **9.** $\sqrt{x+4} \cdot \sqrt{x-4}$
10. $\sqrt{x-3} \cdot \sqrt{2x+4}$ **11.** $\sqrt{2x+5} \cdot \sqrt{x-4}$ **12.** $\sqrt{3a} \cdot \sqrt{3a+2b}$
13. $\sqrt{x} \cdot \sqrt{3x+4y}$ **14.** $\sqrt{a-b} \cdot \sqrt{a+b}$ **15.** $\sqrt{x-3} \cdot \sqrt{x+4}$

Multiply and simplify.

16. $\sqrt{3} \cdot \sqrt{18}$ **17.** $\sqrt{15} \cdot \sqrt{6}$ **18.** $\sqrt{3} \cdot \sqrt{27}$
19. $\sqrt{18} \cdot \sqrt{14x}$ **20.** $\sqrt{7x} \cdot \sqrt{21y}$ **21.** $\sqrt{11} \cdot \sqrt{11x}$
22. $\sqrt{5b} \cdot \sqrt{15b}$ **23.** $\sqrt{6a} \cdot \sqrt{18a}$ **24.** $\sqrt{2t} \cdot \sqrt{2t}$
25. $\sqrt{ab} \cdot \sqrt{ac}$ **26.** $\sqrt{xy} \cdot \sqrt{xz}$ **27.** $\sqrt{2x^2y} \cdot \sqrt{4xy^2}$
28. $\sqrt{15mn^2} \cdot \sqrt{5m^2n}$ **29.** $\sqrt{18x^2y^3} \cdot \sqrt{6xy^4}$ **30.** $\sqrt{12x^3y^2} \cdot \sqrt{8xy}$
31. $\sqrt{50ab} \cdot \sqrt{10a^2b^4}$ **32.** $\sqrt{5a} \cdot \sqrt{20ab}$ **33.** $\sqrt{7a^2b} \cdot \sqrt{42a^3b^2}$
34. $\sqrt{56x^2y^7} \cdot \sqrt{8xy}$ **35.** $\sqrt{10x^6y^3} \cdot \sqrt{2x^5y}$ **36.** $\sqrt{15xy^{12}} \cdot \sqrt{3x^3y^5}$
37. $\sqrt{8xyz^3} \cdot \sqrt{10x^3y^2z}$ **38.** $\sqrt{12x^3y^5z} \cdot \sqrt{5xy^2z}$ **39.** $\sqrt{12x^3} \cdot \sqrt{5x} \cdot \sqrt{45}$
40. $\sqrt{12x^6} \cdot \sqrt{7x^3} \cdot \sqrt{42x}$ **41.** $\sqrt{6x^3} \cdot \sqrt{5x^5} \cdot \sqrt{10x^6}$

B

Multiply and simplify.

42. $(\sqrt{2y})(\sqrt{3})(\sqrt{8y})$ **43.** $\sqrt{a}\left(\sqrt{a^3} - 5\right)$
44. $\sqrt{27(x+1)} \cdot \sqrt{12y(x+1)^2}$ **45.** $\sqrt{18(x-2)} \cdot \sqrt{20(x-2)^3}$
46. $\sqrt{x} \cdot \sqrt{2x} \cdot \sqrt{10x^5}$ **47.** $\sqrt{0.04x^{4n}}$
48. $\sqrt{2^{109}} \cdot \sqrt{x^{306}} \cdot \sqrt{x^{11}}$ **49.** $\sqrt{147} \cdot \sqrt{y^{27}} \cdot \sqrt{x^{315}}$
50. $\sqrt{(x+9)^4} \cdot \sqrt{(x+9)^{99}}$ **51.** $\sqrt{a^2 + 4ab + 4b^2} \cdot \sqrt{(a+2b)^{32}}$
52. $\sqrt{x^{2n}} \cdot \sqrt{y^{2n+1}}$ **53.** $\sqrt{x^{2n}} \cdot \sqrt{x^3y^{3n}} \cdot \sqrt{y^{n+1}}$

54. ***Critical Thinking*** We know that $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ for positive real numbers. Is it also true that $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$? Explain.

Challenge

55. Simplify $\sqrt{y^n}$, given n is an even whole number ≥ 2.

56. Simplify $\sqrt{y^n}$, given n is an odd whole number ≥ 3.

57. Multiply $(x^2 + \sqrt{2}xy + y^2)$ by $(x^2 - \sqrt{2}xy + y^2)$. Use your result to factor $x^8 + y^8$.

Exercises
1. $\sqrt{6}$
2. $\sqrt{15}$
3. 17
4. $5\sqrt{3}$
5. $\sqrt{2x}$
6. $\sqrt{x^2 - 3x}$
7. $\sqrt{10x - 5}$
8. $\sqrt{x^2 + 3x + 2}$
9. $\sqrt{x^2 - 16}$
10. $\sqrt{2x^2 - 2x - 12}$
11. $\sqrt{2x^2 - 3x - 20}$
12. $\sqrt{9a^2 + 6ab}$
13. $\sqrt{3x^2 + 4xy}$
14. $\sqrt{a^2 - b^2}$
15. $\sqrt{x^2 + x - 12}$
16. $3\sqrt{6}$
17. $3\sqrt{10}$
18. 9
19. $6\sqrt{7x}$
20. $7\sqrt{3xy}$
21. $11\sqrt{x}$
22. $5b\sqrt{3}$
23. $6a\sqrt{3}$
24. $2t$
25. $a\sqrt{bc}$
26. $x\sqrt{yz}$
27. $2xy\sqrt{2xy}$
28. $5mn\sqrt{3mn}$
29. $6xy^3\sqrt{3xy}$
30. $4x^2y\sqrt{6y}$
31. $10ab^2\sqrt{5ab}$
32. $10a\sqrt{b}$
33. $7a^2b\sqrt{6ab}$
34. $8xy^4\sqrt{7x}$
35. $2x^5y^2\sqrt{5x}$
36. $3x^2y^8\sqrt{5y}$
37. $4x^2yz^2\sqrt{5y}$
38. $2x^2y^3z\sqrt{15y}$
39. $30x^2\sqrt{3}$
40. $42x^5\sqrt{2}$
41. $10x^7\sqrt{3}$
42. $4y\sqrt{3}$
43. $a^2 - 5\sqrt{a}$
44. $18(x+1)\sqrt{y(x+1)}$
45. $6(x-2)^2\sqrt{10}$

Mixed Review

Simplify. **58.** $m^6 \cdot m^2$ **59.** $(3y^2)^3$ **60.** $(4x^3)(x^2)$ **61.** $(4c^2)(-2c^3)$ *5-2*

Factor. **62.** $a^2 - b^2$ **63.** $144y^2 - 1$ **64.** $4m^2 - 9n^2$ *6-2*

Simplify. **65.** $\frac{-7x}{x+3} - \frac{2x+9}{x+3}$ **66.** $\frac{4}{x} + \frac{3}{x^2}$ **67.** $\frac{x+1}{2} - \frac{x-3}{4}$ *10-4, 10-5*

Find the square roots of each number. **68.** 36 **69.** 121 **70.** 625 *11-1*

Simplify. **71.** $\sqrt{16}$ **72.** $-\sqrt{36}$ **73.** $-\sqrt{225}$ **74.** $\sqrt{81}$ *11-2*

Rational Exponents

Objective: Simplify expressions with rational exponents.

Your work with exponents thus far has been with integer exponents. Exponents can also be rational numbers.

If rational exponents are to follow the same rules as for integer exponents, it must be true that $3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 3^{\frac{1}{2}+\frac{1}{2}} = 3^1 = 3$.

If we define $3^{\frac{1}{2}} = \sqrt{3}$, then our rules will work, since $\sqrt{3} \cdot \sqrt{3} = \sqrt{3^2} = 3$.

Definition

Rational Exponent: $a^{\frac{1}{k}} = \sqrt[k]{a}$ for any natural number k and any a, $a > 0$.

EXAMPLES Simplify.

1 $25^{\frac{1}{2}} = \sqrt{25} = 5$

2 $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

Finding Roots

You can find a cube root or other roots of a number using the y^x key.

To find the cube root of 8, we first write $\sqrt[3]{8}$ using exponential notation.

$\sqrt[3]{8} = 8^{\frac{1}{3}}$

$8^{\frac{1}{3}} \rightarrow$ 8 [y^x] [(] 1 [÷] 3 [)] [=] $\rightarrow 2$

Exercises

Simplify.

1. $16^{\frac{1}{2}}$ **2.** $81^{\frac{1}{2}}$ **3.** $27^{\frac{1}{3}}$ **4.** $125^{\frac{1}{3}}$ **5.** $343^{\frac{1}{3}}$

6. $(9^{\frac{1}{2}})^3$ **7.** $(4^{\frac{1}{2}})^3$ **8.** $(64^{\frac{1}{3}})^2$ **9.** $(27^{\frac{1}{3}})^2$ **10.** $(125^{\frac{1}{3}})^2$

Self-Test On the Web

Check your progress. Look for a self-test at the Prentice Hall Web site. www.phschool.com

California Topic

CA 2.0: Understand and use the operation of raising to a fractional power.

Rational Exponents

Ask students to find the cube roo[t] number with their calculators, an[d] will respond that their calculator [doesn't] have a cube root key. This is a goo[d] to give a practical demonstration [of the] use of rational exponents. Ration[al] exponents allow us to find the cu[be root] of any number. This can also be e[xtended] to finding any root desired, using [$\frac{1}{4}$ to find] the fourth root, $\frac{1}{5}$ to find the fifth and so on.

Exercises

46. $2x^3\sqrt{5x}$
47. $0.2x^{2n}$
48. $2^{54}x^{158}\sqrt{2x}$
49. $7y^{13}x^{157}\sqrt{3xy}$
50. $(x+9)^{51}\sqrt{x+9}$
51. $(a+2b)^{17}$
52. $x^ny^n\sqrt{y}$
53. $x^{n+1}y^{2n}\sqrt{xy}$
54. No. Consider $a = 1$ and $b = 1$. $\sqrt{1} + \sqrt{1} = 2$; $\sqrt{1+1} = \sqrt{2}$
55. $y^{\frac{n}{2}}$
56. $y^{\frac{(n-1)}{2}}\sqrt{y}$
57. $x^4 + y^4$; $(x^4 + \sqrt{2}x^2y^2 + y^4)(x^4 - \sqrt{2}x^2y^2 + y^4)$

Mixed Review

58. m^8
59. $27y^6$
60. $4x^5$
61. $-8c^5$
62. $(a+b)(a-b)$
63. $(12y-1)(12y+1)$
64. $(2m+3n)(2m-3n)$
65. $\frac{-9(x+1)}{x+3}$
66. $\frac{4x+3}{x^2}$
67. $\frac{x+5}{4}$
68. ± 6
69. ± 11
70. ± 25
71. 4
72. -6
73. -15
74. 9

RATIONAL EXPONENTS

Exercises

1. 4
2. 9
3. 3
4. 5
5. 7
6. 27
7. 8
8. 16
9. 9
10. 25

Reading Math

You read $\frac{\sqrt{5}}{7}$ as the quotient of the square root of five and seven.

You read $\frac{\sqrt{5}}{\sqrt{7}}$ as the square root of five divided by the square root of seven.

You read $\sqrt{\frac{5}{7}}$ as the square root of five sevenths.

8 $\frac{\sqrt{5}}{\sqrt{x}} = \frac{\sqrt{5}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$ Multiplying by 1; $\frac{\sqrt{x}}{\sqrt{x}} = 1$

$= \frac{\sqrt{5} \cdot \sqrt{x}}{\sqrt{x} \cdot \sqrt{x}}$

$= \frac{\sqrt{5x}}{x}$

Try This Simplify.

h. $\frac{\sqrt{5}}{\sqrt{7}}$ **i.** $\frac{8}{\sqrt{6}}$ **j.** $\frac{\sqrt{x}}{\sqrt{y}}$

If the radicand is a fraction, it can be simplified by writing it as a division c radicals and proceeding as above.

EXAMPLES Simplify.

9 $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}}$ Writing as a division of radicals

$= \frac{\sqrt{3}}{2}$

10 $\sqrt{\frac{5}{12}} = \frac{\sqrt{5}}{\sqrt{12}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Multiplying by 1; $\frac{\sqrt{3}}{\sqrt{3}} = 1$

$= \frac{\sqrt{15}}{\sqrt{36}}$

$= \frac{\sqrt{15}}{6}$

11 $\sqrt{\frac{2y}{5x^3}} = \frac{\sqrt{2y}}{\sqrt{5x^3}}$

$= \frac{\sqrt{2y}}{\sqrt{5x^3}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}}$ Multiplying by 1; $\frac{\sqrt{5x}}{\sqrt{5x}} = 1$

$= \frac{\sqrt{10yx}}{\sqrt{25x^4}}$

$= \frac{\sqrt{10yx}}{5x^2}$

Try This Simplify.

k. $\sqrt{\frac{3}{7}}$ **l.** $\sqrt{\frac{5}{8}}$ **m.** $\sqrt{\frac{2}{27}}$ **n.** $\sqrt{\frac{5}{2a}}$

o. $\sqrt{\frac{7}{3b^5}}$ **p.** $\sqrt{\frac{x}{18y^3}}$

Try This

h. $\frac{\sqrt{35}}{7}$
i. $\frac{4\sqrt{6}}{3}$
j. $\frac{\sqrt{xy}}{y}$
k. $\frac{\sqrt{21}}{7}$
l. $\frac{\sqrt{10}}{4}$
m. $\frac{\sqrt{6}}{9}$
n. $\frac{\sqrt{10a}}{2a}$
o. $\frac{\sqrt{21b}}{3b^3}$
p. $\frac{\sqrt{2xy}}{6y^2}$

Exercises

1. $\frac{3}{7}$
2. $\frac{4}{5}$
3. $\frac{1}{6}$
4. $\frac{1}{2}$
5. $-\frac{4}{9}$
6. $\frac{-5}{7}$
7. $\frac{8}{17}$
8. $\frac{9}{19}$
9. $-\frac{3}{10}$
10. $-\frac{7}{10}$
11. $\frac{3}{5}$
12. $\frac{5}{3}$
13. 3
14. 2
15. 2
16. 6
17. $\sqrt{5}$
18. $\sqrt{6}$
19. $\frac{1}{5}$
20. $\frac{1}{4}$
21. $\frac{2}{5}$
22. $\frac{3}{4}$
23. 2
24. 3
25. $3y$
26. $4x$
27. $x^2\sqrt{5}$
28. $a^2\sqrt{6}$
29. $\frac{1}{3}\sqrt{21}$
30. $\frac{1}{5}\sqrt{10}$
31. $\frac{3}{4}\sqrt{2}$
32. $\frac{2}{9}\sqrt{3}$
33. $\frac{1}{5}\sqrt{10}$
34. $\frac{1}{7}\sqrt{14}$
35. $\frac{1}{4}\sqrt{6}$
36. $\frac{1}{4}\sqrt{14}$
37. $\frac{1}{6}\sqrt{21}$

500 Chapter 11 *Radical Expressions and Equations*

11-5 Exercises

A

Simplify.

1. $\sqrt{\frac{9}{49}}$ **2.** $\sqrt{\frac{16}{25}}$ **3.** $\sqrt{\frac{1}{36}}$ **4.** $\sqrt{\frac{1}{4}}$

5. $-\sqrt{\frac{16}{81}}$ **6.** $-\sqrt{\frac{25}{49}}$ **7.** $\sqrt{\frac{64}{289}}$ **8.** $\sqrt{\frac{81}{361}}$

9. $-\sqrt{\frac{9}{100}}$ **10.** $-\sqrt{\frac{49}{100}}$ **11.** $\sqrt{\frac{27}{75}}$ **12.** $\sqrt{\frac{50}{18}}$

Divide and simplify.

13. $\frac{\sqrt{18}}{\sqrt{2}}$ **14.** $\frac{\sqrt{20}}{\sqrt{5}}$ **15.** $\frac{\sqrt{60}}{\sqrt{15}}$ **16.** $\frac{\sqrt{108}}{\sqrt{3}}$

17. $\frac{\sqrt{75}}{\sqrt{15}}$ **18.** $\frac{\sqrt{18}}{\sqrt{3}}$ **19.** $\frac{\sqrt{3}}{\sqrt{75}}$ **20.** $\frac{\sqrt{3}}{\sqrt{48}}$

21. $\frac{\sqrt{12}}{\sqrt{75}}$ **22.** $\frac{\sqrt{18}}{\sqrt{32}}$ **23.** $\frac{\sqrt{8x}}{\sqrt{2x}}$ **24.** $\frac{\sqrt{18b}}{\sqrt{2b}}$

25. $\frac{\sqrt{63y^3}}{\sqrt{7y}}$ **26.** $\frac{\sqrt{48x^3}}{\sqrt{3x}}$ **27.** $\frac{\sqrt{15x^5}}{\sqrt{3x}}$ **28.** $\frac{\sqrt{30a^5}}{\sqrt{5a}}$

29. $\frac{\sqrt{7}}{\sqrt{3}}$ **30.** $\frac{\sqrt{2}}{\sqrt{5}}$ **31.** $\frac{\sqrt{9}}{\sqrt{8}}$ **32.** $\frac{\sqrt{4}}{\sqrt{27}}$

Simplify.

33. $\sqrt{\frac{2}{5}}$ **34.** $\sqrt{\frac{2}{7}}$ **35.** $\sqrt{\frac{3}{8}}$ **36.** $\sqrt{\frac{7}{8}}$

37. $\sqrt{\frac{7}{12}}$ **38.** $\sqrt{\frac{1}{12}}$ **39.** $\sqrt{\frac{1}{18}}$ **40.** $\sqrt{\frac{5}{18}}$

41. $\sqrt{\frac{1}{2}}$ **42.** $\sqrt{\frac{1}{3}}$ **43.** $\sqrt{\frac{8}{3}}$ **44.** $\sqrt{\frac{12}{5}}$

45. $\sqrt{\frac{3}{x}}$ **46.** $\sqrt{\frac{2}{x}}$ **47.** $\sqrt{\frac{x}{y}}$ **48.** $\sqrt{\frac{a}{b}}$

49. $\sqrt{\frac{x^2}{18}}$ **50.** $\sqrt{\frac{x^2}{20}}$ **51.** $\sqrt{\frac{6c}{2d^3}}$ **52.** $\sqrt{\frac{x}{8y^7}}$

53. $\frac{\sqrt{2}}{\sqrt{5}}$ **54.** $\frac{\sqrt{3}}{\sqrt{2}}$ **55.** $\frac{2}{\sqrt{2}}$ **56.** $\frac{3}{\sqrt{3}}$

57. $\frac{\sqrt{48}}{\sqrt{32}}$ **58.** $\frac{\sqrt{56}}{\sqrt{40}}$ **59.** $\frac{\sqrt{450}}{\sqrt{18}}$ **60.** $\frac{\sqrt{224}}{\sqrt{14}}$

61. $\frac{\sqrt{3}}{\sqrt{x}}$ **62.** $\frac{\sqrt{2}}{\sqrt{y}}$ **63.** $\frac{4y}{\sqrt{3}}$ **64.** $\frac{8x}{\sqrt{5}}$

65. $\frac{\sqrt{a^3}}{\sqrt{8}}$ **66.** $\frac{\sqrt{x^3}}{\sqrt{27}}$ **67.** $\frac{\sqrt{56}}{\sqrt{12x}}$ **68.** $\frac{\sqrt{45}}{\sqrt{8a}}$

69. $\frac{\sqrt{27c}}{\sqrt{32c^3}}$ **70.** $\frac{\sqrt{7x^3}}{\sqrt{12x}}$ **71.** $\frac{\sqrt{y^5}}{\sqrt{xy^2}}$ **72.** $\frac{\sqrt{x^3}}{\sqrt{xy}}$

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

3. PRACTICE/ASSESS

LESSON QUIZ

1. Simplify $\frac{\sqrt{100}}{\sqrt{64}}$. $\frac{5}{4}$
2. Divide and simplify. $\frac{\sqrt{27x^3}}{\sqrt{3x}}$ $3x$
3. Rationalize the denominator. $\frac{\sqrt{7x}}{\sqrt{5}}$ $\frac{\sqrt{35x}}{5}$

Assignment Guide

To provide flexible scheduling lesson can be split into parts.

1 Core 1–20
Extension 80–83

2 Core 21–72
Extension 73–79, 84

Use Mixed Review to maintain

38. $\frac{1}{6}\sqrt{3}$
39. $\frac{1}{6}\sqrt{2}$
40. $\frac{1}{6}\sqrt{10}$
41. $\frac{1}{2}\sqrt{2}$
42. $\frac{1}{3}\sqrt{3}$
43. $\frac{2}{3}\sqrt{6}$
44. $\frac{2}{5}\sqrt{15}$
45. $\frac{1}{x}\sqrt{3x}$
46. $\frac{1}{x}\sqrt{2x}$
47. $\frac{1}{y}\sqrt{xy}$
48. $\frac{1}{b}\sqrt{ab}$
49. $\frac{x\sqrt{2}}{6}$
50. $\frac{x\sqrt{5}}{10}$
51. $\frac{\sqrt{3cd}}{d^2}$
52. $\frac{\sqrt{2xy}}{4y^4}$
53. $\frac{\sqrt{10}}{5}$
54. $\frac{\sqrt{6}}{2}$
55. $\sqrt{2}$
56. $\sqrt{3}$
57. $\frac{\sqrt{6}}{2}$
58. $\frac{\sqrt{35}}{5}$
59. 5
60. 4
61. $\frac{\sqrt{3x}}{x}$
62. $\frac{\sqrt{2y}}{y}$
63. $\frac{4y\sqrt{3}}{3}$
64. $\frac{8x\sqrt{5}}{5}$
65. $\frac{a\sqrt{2a}}{4}$
66. $\frac{x\sqrt{3x}}{9}$
67. $\frac{\sqrt{42x}}{3x}$
68. $\frac{3\sqrt{10a}}{4a}$
69. $\frac{3\sqrt{6}}{8c}$
70. $\frac{x\sqrt{21}}{6}$
71. $\frac{y\sqrt{xy}}{x}$
72. $\frac{x\sqrt{y}}{y}$

B

Rationalize the denominator.

73. $\frac{\sqrt{2}}{3\sqrt{3}}$ **74.** $\frac{3\sqrt{6}}{6\sqrt{2}}$ **75.** $\frac{5\sqrt{2}}{3\sqrt{5}}$

76. $\frac{3\sqrt{15}}{5\sqrt{32}}$ **77.** $\frac{4\sqrt{\frac{6}{7}}}{\sqrt{\frac{12}{63}}}$ **78.** $\frac{\sqrt{\frac{2}{3}}}{\sqrt{\frac{3}{2}}}$

This pendulum is 1 ft long. How long is its period? (See Exercise 84.)

79. ***Critical Thinking*** Rationalize the denominator of $\frac{a\sqrt{b}}{b\sqrt{a}}$.

Challenge

Multiply.

80. $(\sqrt{5} + 7)(\sqrt{5} - 7)$

81. $(1 + \sqrt{5})(1 - \sqrt{5})$

82. $(\sqrt{6} - \sqrt{3})(\sqrt{6} + \sqrt{3})$

83. $(\sqrt{3} + \sqrt{2})(\sqrt{3} + \sqrt{2})$

84. The period T of a pendulum is the time it takes for a pendulum of length L to move from one side to the other and back.
A formula for the period is $T = 2\pi\sqrt{\frac{L}{32}}$ where T is in seconds and L is in feet. Use 3.14 for π.

a. Find the periods of pendulums of lengths 2 ft, 8 ft, 64 ft, and 100 ft.

b. Find the period of a pendulum of length $\frac{2}{3}$ in.

c. The pendulum of a grandfather clock is $\frac{32}{\pi^2}$ feet long. How long does it take to swing from one side to the other?

Mixed Review

Identify the rational and irrational numbers.

85. $\sqrt{7}$ **86.** $\sqrt{9}$ **87.** $\sqrt{135}$ **88.** $\sqrt{16}$

89. $\sqrt{144}$ **90.** $\sqrt{220}$ **91.** $\sqrt{0}$ *11-1*

Factor. **92.** $16a^2 - 25c^4$ **93.** $x^2 - 2x - 15$

94. $5m^2 - 30m + 45$ **95.** $2am + bm - 6an - 3bn$ *6-2, 6-4, 6-5, 6-7*

Solve. **96.** $\frac{7}{2x} + \frac{2}{x} = 1$ **97.** $\frac{12}{x + 4} = \frac{3}{x - 2}$ **98.** $\frac{a + 1}{4a - 4} = \frac{1}{2}$ *10-6*

Determine the replacements for the variables that give real numbers.

99. $\sqrt{3x}$ **100.** $\sqrt{2x^2}$ **101.** $\sqrt{x - 2}$ **102.** $\sqrt{2x + 5}$ *11-2*

Exercises

73. $\frac{\sqrt{6}}{9}$
74. $\frac{\sqrt{3}}{2}$
75. $\frac{\sqrt{10}}{3}$
76. $\frac{3\sqrt{30}}{40}$
77. $6\sqrt{2}$
78. $\frac{2}{3}$
79. $\frac{\sqrt{ab}}{b}$
80. -44
81. -4
82. 3
83. $5 + 2\sqrt{6}$
84a. 1.57 s, 3.14 s, 8.88 s, 11.10 s
b. 0.262 s
c. 1 s

Photo caption: 1.11 seconds

Mixed Review

85. Irrational
86. Rational
87. Irrational
88. Rational
89. Rational
90. Irrational
91. Rational
92. $(4a + 5c^2)(4a - 5c^2)$
93. $(x + 3)(x - 5)$
94. $5(m - 3)^2$
95. $(m - 3n)(2a + b)$
96. $\frac{11}{2}$
97. 4
98. 3
99. $x \geq 0$
100. Any real number
101. $x \geq 2$
102. $x \geq -\frac{5}{2}$

CA: Applying 1.0, 2.0

1-6

TIME-FOCUS METER

OCUS

FIVE MINUTES

nplify $\frac{\sqrt{9}}{\sqrt{16}}$.

nplify $\frac{\sqrt{50x^3}}{\sqrt{2x}}$.

tionalize the denominator. $\frac{\sqrt{3x}}{\sqrt{5}}$

5x

EACH the Mathematics

asize that an expression such as $\sqrt{2}$ al number and can be manipulated ny other number. $3\sqrt{5} + 4\sqrt{5}$ can ded just as can $3x + 4x$. Stress that $\sqrt{2}$ cannot be simplified, just as cannot be simplified.

uestions

s $\sqrt{5} + \sqrt{6} = \sqrt{11}$?

s $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$?

s $\sqrt{3} + 2\sqrt{3} = 3\sqrt{6}$?

board Examples

r subtract.

$\sqrt{7} + 6\sqrt{7}$

$+ 6)\sqrt{7} = 8\sqrt{7}$

$12 - \sqrt{3}$

$\sqrt{3} - \sqrt{3} = \sqrt{3}$

$9y + \sqrt{16y}$

$\sqrt{y} + 4\sqrt{y} = 7\sqrt{y}$

$a^5 + 2a^4 + \sqrt{a+2}$

$= \sqrt{a^4(a+2)} + \sqrt{a+2}$

$= a^2\sqrt{a+2} + \sqrt{a+2}$

$= (a^2+1)\sqrt{a+2}$

$7 + \frac{\sqrt{1}}{\sqrt{7}}$

$= \sqrt{7} + \frac{\sqrt{1}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$= \sqrt{7} + \frac{\sqrt{7}}{7}$

$= \sqrt{7}\left(1 + \frac{1}{7}\right)$

$= \frac{8}{7}\sqrt{7}$

Applying CA 1.0, 2.0: Use arithmetic properties of real numbers; use the operation of taking a root.

11-6 Addition and Subtraction

Objective: Add and subtract radical expressions.

What You'll Learn

1 To add and subtract radical expressions

. . . And Why

To understand how the rules of addition and subtraction apply to radical expressions

When we have radical expressions with the same radicands we can simplify using the distributive property.

EXAMPLE 1 Add.

$$3\sqrt{5} + 4\sqrt{5} = (3+4)\sqrt{5} \quad \text{Using the distributive property}$$
$$= 7\sqrt{5}$$

Sometimes we need to simplify the radicand before adding or subtracting.

EXAMPLES Add or subtract.

2 $\sqrt{2} - \sqrt{8} = \sqrt{2} - \sqrt{4 \cdot 2}$
$= \sqrt{2} - 2\sqrt{2}$ Simplifying
$= (1-2)\sqrt{2}$ Using the distributive property
$= -\sqrt{2}$

3 $\sqrt{x} + \sqrt{4x} = \sqrt{x} + 2\sqrt{x} = 3\sqrt{x}$

4 $\sqrt{x^3 - x^2} + \sqrt{4x - 4} = \sqrt{x^2(x-1)} + \sqrt{4(x-1)}$ Factoring radicands
$= \sqrt{x^2}\sqrt{x-1} + \sqrt{4}\sqrt{x-1}$
$= x\sqrt{x-1} + 2\sqrt{x-1}$
$= (x+2)\sqrt{x-1}$ Using the distributive property

Try This Add or subtract.

a. $3\sqrt{2} + 9\sqrt{2}$ **b.** $8\sqrt{5} - 3\sqrt{5}$ **c.** $2\sqrt{10} - 7\sqrt{40}$
d. $\sqrt{24y} + \sqrt{54y}$ **e.** $\sqrt{9x+9} - \sqrt{4x+4}$

Sometimes after rationalizing denominators, we can factor and combine expressions.

EXAMPLE 5

$$\sqrt{3} + \sqrt{\tfrac{1}{3}} = \sqrt{3} + \frac{\sqrt{1}}{\sqrt{3}}$$
$$= \sqrt{3} + \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \sqrt{3} + \frac{\sqrt{3}}{3}$$
$$= \frac{4}{3}\sqrt{3}$$

Try This

a. $12\sqrt{2}$
b. $5\sqrt{5}$
c. $-12\sqrt{10}$
d. $5\sqrt{6y}$
e. $\sqrt{x+1}$
f. $\frac{3}{2}\sqrt{2}$
g. $\frac{2\sqrt{15}}{15}$
h. $2\sqrt{x}$

Exercises

1. $7\sqrt{2}$
2. $11\sqrt{3}$
3. $4\sqrt{5}$
4. $3\sqrt{2}$
5. $13\sqrt{x}$
6. $12\sqrt{y}$
7. $-2\sqrt{x}$
8. $-8\sqrt{a}$
9. $25\sqrt{2}$
10. $8\sqrt{3}$
11. $\sqrt{3}$
12. $32\sqrt{2}$
13. $\sqrt{5}$
14. $\sqrt{3}$

Try This Add or subtract.

f. $\sqrt{2} + \sqrt{\frac{1}{2}}$ g. $\sqrt{\frac{5}{3}} - \sqrt{\frac{3}{5}}$ h. $\frac{x}{\sqrt{x}} + \sqrt{x}$

11-6 Exercises

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site. www.phschool.com

A

Add or subtract.

1. $3\sqrt{2} + 4\sqrt{2}$
2. $8\sqrt{3} + 3\sqrt{3}$
3. $7\sqrt{5} - 3\sqrt{5}$
4. $8\sqrt{2} - 5\sqrt{2}$
5. $6\sqrt{x} + 7\sqrt{x}$
6. $9\sqrt{y} + 3\sqrt{y}$
7. $9\sqrt{x} - 11\sqrt{x}$
8. $6\sqrt{a} - 14\sqrt{a}$
9. $5\sqrt{8} + 15\sqrt{2}$
10. $3\sqrt{12} + 2\sqrt{3}$
11. $\sqrt{27} - 2\sqrt{3}$
12. $7\sqrt{50} - 3\sqrt{2}$
13. $\sqrt{45} - \sqrt{20}$
14. $\sqrt{27} - \sqrt{12}$
15. $\sqrt{72} + \sqrt{98}$
16. $\sqrt{45} + \sqrt{80}$
17. $2\sqrt{12} + \sqrt{27} - \sqrt{48}$
18. $9\sqrt{8} - \sqrt{72} + \sqrt{98}$
19. $3\sqrt{18} - 2\sqrt{32} - 5\sqrt{50}$
20. $\sqrt{18} - 3\sqrt{8} + \sqrt{50}$
21. $2\sqrt{27} - 3\sqrt{48} + 2\sqrt{18}$
22. $3\sqrt{48} - 2\sqrt{27} - 2\sqrt{18}$
23. $\sqrt{4x} + \sqrt{81x^3}$
24. $\sqrt{12x^2} + \sqrt{27}$
25. $\sqrt{27} - \sqrt{12x^2}$
26. $\sqrt{81x^3} - \sqrt{4x}$
27. $\sqrt{8x + 8} + \sqrt{2x + 2}$
28. $\sqrt{12x + 12} + \sqrt{3x + 3}$
29. $\sqrt{x^5 - x^2} + \sqrt{9x^3 - 9}$
30. $\sqrt{16x - 16} + \sqrt{25x^3 - 25x^2}$
31. $3x\sqrt{y^3x} - x\sqrt{yx^3} + y\sqrt{y^3x}$
32. $4a\sqrt{a^2b} + a\sqrt{a^2b^3} - 5\sqrt{b^3}$
33. $\sqrt{4(a + b)} - \sqrt{(a + b)^3}$
34. $\sqrt{x^2y} + \sqrt{4x^2y} + \sqrt{9y} - \sqrt{y^3}$
35. $\sqrt{3} - \sqrt{\frac{1}{3}}$
36. $\sqrt{2} - \sqrt{\frac{1}{2}}$
37. $5\sqrt{2} + 3\sqrt{\frac{1}{2}}$
38. $4\sqrt{3} + 2\sqrt{\frac{1}{3}}$
39. $\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{6}}$
40. $\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{8}}$
41. $\sqrt{\frac{1}{12}} - \sqrt{\frac{1}{27}}$
42. $\sqrt{\frac{5}{6}} - \sqrt{\frac{6}{5}}$

B

43. ***Error Analysis*** Three students were asked to simplify $\sqrt{10} + \sqrt{50}$. Their answers were $\sqrt{10}(1 + \sqrt{5})$, $\sqrt{10} + 5\sqrt{2}$, and $\sqrt{2}(5 + \sqrt{5})$.
    a. Which, if any, is incorrect?
    b. Which is in simplest form?

Try This

a. $12\sqrt{2}$
b. $5\sqrt{5}$
c. $-12\sqrt{10}$
d. $5\sqrt{6y}$
e. $\sqrt{x} + 1$
f. $\frac{3}{2}\sqrt{2}$
g. $\frac{2\sqrt{15}}{15}$
h. $2\sqrt{x}$

3. PRACTICE/ASSESS

LESSON QUIZ

Add or subtract.

1. $8\sqrt{11} + 3\sqrt{11}$
   $11\sqrt{11}$
2. $7\sqrt{45} - 4\sqrt{5}$
   $17\sqrt{5}$
3. $3\sqrt{4a} + 5\sqrt{9a^3}$
   $(6 + 15a)\sqrt{a}$

Assignment Guide

▼ Core 1–52
Extension 53–57

Use Mixed Review to maintain

Exercises

1. $7\sqrt{2}$
2. $11\sqrt{3}$
3. $4\sqrt{5}$
4. $3\sqrt{2}$
5. $13\sqrt{x}$
6. $12\sqrt{y}$
7. $-2\sqrt{x}$
8. $-8\sqrt{a}$
9. $25\sqrt{2}$
10. $8\sqrt{3}$
11. $\sqrt{3}$
12. $32\sqrt{2}$
13. $\sqrt{5}$
14. $\sqrt{3}$
15. $13\sqrt{2}$
16. $7\sqrt{5}$
17. $3\sqrt{3}$
18. $19\sqrt{2}$
19. $-24\sqrt{2}$
20. $2\sqrt{2}$
21. $6\sqrt{2} - 6\sqrt{3}$
22. $6\sqrt{3} - 6\sqrt{2}$
23. $(2 + 9x)\sqrt{x}$
24. $(2x + 3)\sqrt{3}$
25. $(3 - 2x)\sqrt{3}$
26. $(9x - 2)\sqrt{x}$
27. $3\sqrt{2x + 2}$
28. $3\sqrt{3x + 3}$
29. $(x + 3)\sqrt{x^3 - 1}$
30. $(4 + 5x)\sqrt{x - 1}$
31. $(-x^2 + 3xy + y^2)\sqrt{xy}$
32. $(4a^2 + a^2b - 5b)\sqrt{b}$
33. $(2 - a - b)\sqrt{a + b}$
34. $(3x + 3 - y)\sqrt{y}$
35. $\frac{2\sqrt{3}}{3}$
36. $\frac{\sqrt{2}}{2}$
37. $\frac{13\sqrt{2}}{2}$
38. $\frac{14\sqrt{3}}{3}$
39. $\frac{\sqrt{6}}{6}$
40. $\frac{\sqrt{2}}{4}$
41. $\frac{\sqrt{3}}{18}$
42. $\frac{-\sqrt{30}}{30}$

43a. None
  b. $\sqrt{10} + 5\sqrt{2}$

ATICNE

14. $4\sqrt{5}$
15. $16\sqrt{2}$
16. $-\frac{4\sqrt{6}}{5}$
17. $(b^3+ab+a)\sqrt{a}$
18. $11\sqrt{3}-10\sqrt{2}$
19. 0
20. $37xy\sqrt{3x}$
21. $\frac{x+1}{x}\sqrt{x}$
22. $x=0$ or $y=0$
23. $\frac{\sqrt{30}}{12}$
24. $\left[\frac{2}{b}-\frac{2}{a^2}+\frac{5a}{4}\right]\sqrt{2ab}$
25. a. $\sqrt{5}, 3$
b. $\sqrt{13}, 5$
c. $\sqrt{17}, 5$
d. $5, 7$
e. $\sqrt{10}, 4$
f. $2\sqrt{5}, 6$
26. $-5-5\sqrt{2}$
27. $\frac{24+3\sqrt{3}+8\sqrt{2}+\sqrt{6}}{7}$

[redacted] Mixed Review

[redacted] 58. $5|y|$
59. 6
[redacted] 60. $|x+2|$
61. $|y+5|$
[redacted] 62. $5\sqrt{3}$
63. $4\sqrt{3}$
64. 11
65. $\sqrt{a^2-c^2}$
66. $2\sqrt{5}$
67. $4x\sqrt{y}$

Practice Multiple Choice

Choose the best answer.

1. Which of the following statements is true about converses?

The converse of a statement . . .

A is always true.

B is true when the statement is true.

C may be true or false when the statement is true.

D is false when the statement is true.

2. Simplify the radical expression.

$\sqrt{\frac{72x^7}{125x^4}}$

F $\frac{6|x|\sqrt{10x}}{25}$

G $\frac{6}{5}\sqrt{\frac{2x^3}{5}}$

H $\frac{6}{5}x^{\sqrt{3}}\sqrt{0.4}$

J $\sqrt{0.576x^{11}}$

1. C; Algebra 24.0
2. F; Algebra 2.0

Add or subtract.

44. $\sqrt{125}-\sqrt{45}+2\sqrt{5}$

45. $3\sqrt{\frac{1}{2}}+\frac{5}{2}\sqrt{18}+\sqrt{98}$

46. $\frac{3}{5}\sqrt{24}+\frac{2}{5}\sqrt{150}-\sqrt{96}$

47. $\sqrt{ab^6}+b\sqrt{a^3}+a\sqrt{a}$

48. $\frac{1}{3}\sqrt{27}+\sqrt{8}+\sqrt{300}-\sqrt{18}-\sqrt{162}$

49. $x\sqrt{2y}-\sqrt{8x^2y}+\frac{x}{3}\sqrt{18y}$

50. $7x\sqrt{12xy^2}-9y\sqrt{27x^3}+5\sqrt{300x^3y^2}$

51. $\sqrt{x}+\sqrt{\frac{1}{x}}$

52. *Critical Thinking* You know that $\sqrt{x^2+y^2}=\sqrt{x^2}+\sqrt{y^2}$ is not true for *all* real numbers. For what numbers is it true?

Challenge

Add or subtract. Simplify when possible.

53. $5\sqrt{\frac{3}{10}}+2\sqrt{\frac{5}{6}}-6\sqrt{\frac{15}{32}}$

54. $2\sqrt{\frac{2a}{b}}-4\sqrt{\frac{b}{2a^3}}+5\sqrt{\frac{1}{8}a^3b}$

55. Evaluate for $a=1, b=3, c=2, d=4$.

a. $\sqrt{a^2+c^2}, \sqrt{a^2}+\sqrt{c^2}$
b. $\sqrt{b^2+c^2}, \sqrt{b^2}+\sqrt{c^2}$
c. $\sqrt{a^2+d^2}, \sqrt{a^2}+\sqrt{d^2}$
d. $\sqrt{b^2+d^2}, \sqrt{b^2}+\sqrt{d^2}$
e. $\sqrt{a^2+b^2}, \sqrt{a^2}+\sqrt{b^2}$
f. $\sqrt{c^2+d^2}, \sqrt{c^2}+\sqrt{d^2}$

Binomial pairs such as $1+\sqrt{2}$ and $1-\sqrt{2}$ are called **conjugates.** We can use conjugates to rationalize binomial denominators containing radicals. Rationalize each denominator.

56. $\frac{5}{1-\sqrt{2}}$

57. $\frac{8+\sqrt{3}}{3-\sqrt{2}}$

Mixed Review

Simplify. **58.** $\sqrt{25y^2}$ **59.** $\sqrt{(-6)^2}$
60. $\sqrt{(x+2)^2}$ **61.** $\sqrt{y^2+10y+25}$ *11-2*

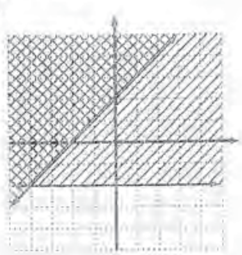
Multiply. **62.** $\sqrt{5}\sqrt{15}$ **63.** $\sqrt{6}\sqrt{8}$
64. $\sqrt{11}\sqrt{11}$ **65.** $\sqrt{a+c}\sqrt{a-c}$ *11-4*

Factor and simplify. **66.** $\sqrt{20}$ **67.** $\sqrt{16x^2y}$ **68.** $\sqrt{x^3-4x^2+4x}$ *11-3*

Solve by graphing. **69.** $x+2\le y$, $y\ge -2$ **70.** $x+y\le 1$, $x+y\ge 1$ **71.** $y-x\le 1$, $2y+x\le 2$ *9-6*

68. $(x-2)\sqrt{x}$

69.



70.

71.

ew

Teach the Mathematics (continued)

ɟ the length of the leg of the right ngle with leg 6 and hypotenuse 11.

$+ 6^2 = 11^2$

$a^2 = 11^2 - 6^2$

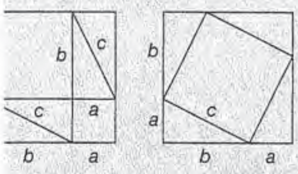
$a^2 = 85$

$a = \sqrt{85} \approx 9.220$

ıching Transparency T46 to present oard Example 1.

N ENRICHMENT

ıching Transparency T47 if you present this informal proof of hagorean theorem.



o large squares have equal areas. ıreas remain after removing the ıht triangles from each large The remaining area in the left is $a^2 + b^2$. The remaining area in ıt square is c^2. Therefore, $= c^2$.

ACTICE/ASSESS

N QUIZ

l the hypotenuse of the right ngle with sides $\sqrt{8}$ and 1.

l the leg of the right triangle with 2 and hypotenuse $\sqrt{13}$.

gnment Guide

ore 1–26

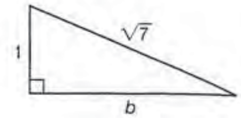
xtension 27–31

Mixed Review to maintain skills.

EXAMPLE 2

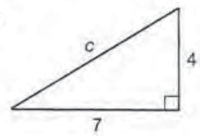
Find the length of the leg of this right triangle to the nearest thousandth.

$$1^2 + b^2 = (\sqrt{7})^2$$
$$1 + b^2 = 7$$
$$b^2 = 7 - 1$$
$$b^2 = 6$$
$$b = \sqrt{6}$$
$$b \approx 2.449$$

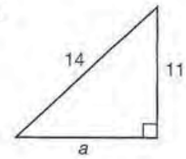


Try This Find the missing length in each right triangle.

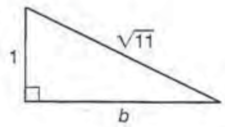
a.



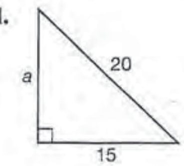
b.



c.



d.



Extra Help On the Web

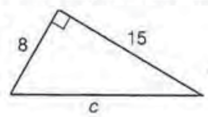
Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

11-7 Exercises

A

Find the length of the third side of each right triangle.

1.



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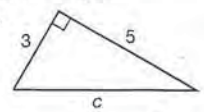

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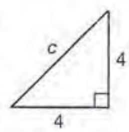

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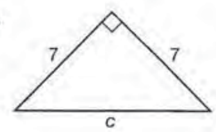
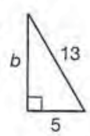
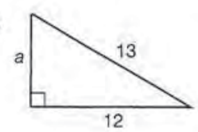


5.



6.



Try This

a. $\sqrt{65} \approx 8.062$

b. $\sqrt{75} \approx 8.660$

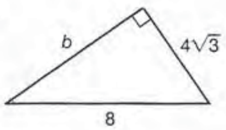
c. $\sqrt{10} \approx 3.162$

d. $\sqrt{175} = 5\sqrt{7} \approx 13.229$

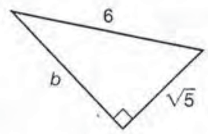
Exercises

1. 17
2. $\sqrt{34}$ or 5.831
3. $4\sqrt{2}$ or 5.657
4. $7\sqrt{2}$ or 9.899
5. 12
6. 5

7.



8.



Find the length of the side not given for a right triangle with hypotenuse c and legs a and b.

9. $a = 10, b = 24$
10. $a = 5, b = 12$
11. $a = 9, c = 15$
12. $a = 18, c = 30$
13. $b = 1, c = \sqrt{5}$
14. $b = 1, c = \sqrt{2}$
15. $a = 1, c = \sqrt{3}$
16. $a = \sqrt{3}, b = \sqrt{5}$
17. $c = 10, b = 5\sqrt{3}$
18. $a = 3\sqrt{3}, c = 5\sqrt{3}$

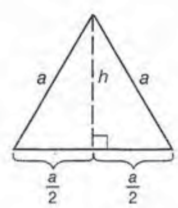
19. ***Error Analysis*** A student found the length of the hypotenuse. What error did the student make?

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 8^2$$
$$\sqrt{c^2} = \sqrt{5^2 + 8^2}$$
$$c = 5 + 8$$
$$c = 13$$

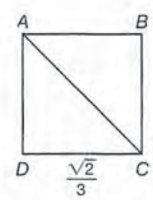
B

An equilateral triangle is shown to the right.

20. Find an expression for height h in terms of a.
21. Find an expression for area A in terms of a.
22. Find an expression for area A in terms of h.



23. **TEST PREP** Which is longest?
    A. the diagonal of a square with 6 cm sides
    B. the diagonal of a rectangle with length 7 cm and width 5 cm
    C. the hypotenuse of a right triangle with legs 4 cm and 9 cm long
    D. the leg of a right triangle with the other leg 10 cm and the hypotenuse 15 cm long



24. Figure $ABCD$ is a square. Find AC.
25. ***Critical Thinking*** A right triangle has sides whose lengths are consecutive integers. Find the lengths of the sides.

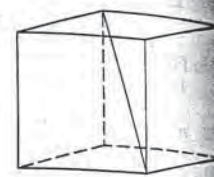
Exercises

7. 4
8. $\sqrt{31}$ or 5.568
9. 26
10. 13
11. 12
12. 24
13. 2
14. 1
15. $\sqrt{2}$
16. $2\sqrt{2}$
17. 5
18. $b = 4\sqrt{3}$
19. $\sqrt{5^2 + 8^2} = \sqrt{25 + 64}$ $= \sqrt{89}$, not $5 + 8 = 13$.
20. $\frac{a}{2}\sqrt{3}$
21. $\frac{a^2}{4}\sqrt{3}$
22. $A = \frac{h^2\sqrt{3}}{3}$
23. D
24. $\frac{2}{3}$
25. 3, 4, 5

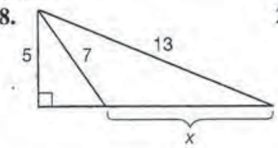
26. Suppose the length of a hypotenuse is $\sqrt{18}$. Find two pairs of values for the lengths of the legs.

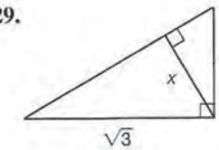
Challenge

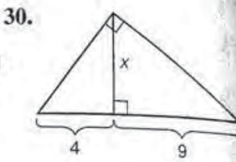
27. Find the length of the diagonal of a cube with side length 10 cm.



Find x.

28.


29.


30.


31. ***Mathematical Reasoning*** The density property of rational numbers states that no matter how close two rational numbers are, there is a rational number between them.
   - **a.** Find a real number between $\frac{3}{7}$ and $\frac{4}{7}$.
   - **b.** Find a real number between –0.23 and –0.24.
   - **c.** Find a real number between $\sqrt{2}$ and $\sqrt{3}$.
   - **d.** Find a real number between $\sqrt{10}$ and $\sqrt{11}$.
   - **e.** Do you think the density property holds for real numbers? Justify your reasoning.

Mixed Review

Simplify. **32.** $-\sqrt{169}$ **33.** $\sqrt{(x+3)^2}$ **34.** $\sqrt{25y}$ **35.** $\sqrt{32a^2}$

36. $\sqrt{x^7}$ **37.** $\sqrt{a^{20}}$ **38.** $\sqrt{(x+5)^4}$ **39.** $\sqrt{(x-3)^3}$

40. $\sqrt{12a^5}$ **41.** $\sqrt{216x^4}$

42. $\sqrt{12x^3y}$ **43.** $\sqrt{18a^3b^4}$ *11-2, 11-3*

Solve. **44.** $\frac{3}{2x} + \frac{1}{2} = \frac{10}{4x}$ **45.** $\frac{x}{4} - \frac{x}{12} = \frac{1}{2}$ **46.** $x + \frac{12}{x} = 7$ *10-6*

47. The product of two consecutive positive integers is 210. Find the numbers.

48. The sum of two numbers is 7. The difference of the two numbers is −17. Find the numbers. *8-6*

Exercises

26. Answers may vary. Samples: $\sqrt{12}$ and $\sqrt{6}$, 2 and $\sqrt{14}$
27. $10\sqrt{3}$
28. $12 - 2\sqrt{6} \approx 7.1$
29. $\frac{\sqrt{3}}{2} \approx 0.87$
30. 6
31. Answers may vary. Samples given.
   a. $\frac{1}{2}$
   b. −0.237
   c. $\sqrt{2.4}$
   d. $\sqrt{10.79}$
   e. Yes. The average of two numbers will always be another real number between the two.

Mixed Review

32. −13
33. $|x+3|$
34. $5\sqrt{y}$
35. $4|a|\sqrt{2}$
36. $x^3\sqrt{x}$
37. a^{10}
38. $(x+5)^2$
39. $(x-3)\sqrt{x-3}$
40. $2a^2\sqrt{3a}$
41. $6x^2\sqrt{6}$
42. $2x\sqrt{3xy}$
43. $3ab^2\sqrt{2a}$
44. 2
45. 3
46. 3, 4
47. 14, 15
48. −5, 12

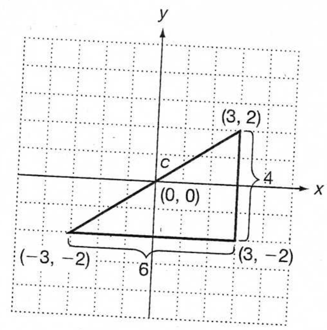
The Distance Formula

Objective: Find the distance between two points on the coordinate plane.

The **distance formula** is based on the Pythagorean theorem and is used to find the distance between any two points in the plane if the coordinates of the points are known.

EXAMPLE 1 Find the distance between the points (3, 2) and (−3, −2).

We plot these points on a coordinate plane and construct a triangle as shown. The distance between the points (3, 2) and (−3, −2) is the length of the hypotenuse of this triangle.



We can use the Pythagorean theorem to find this length.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 6^2 + 4^2 \\ &= 36 + 16 \\ &= 52 \\ c &= \pm\sqrt{52} \end{aligned}$$

Since distance cannot be negative, the distance is $\sqrt{52} \approx 7.2$.

The distance formula, which is derived from the Pythagorean theorem, can be used to compute the distance between two points.

Distance Formula

The distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

EXAMPLE 2 Find the distance between (2, 2) and (5, 6).

$$\begin{aligned} d &= \sqrt{(2 - 5)^2 + (2 - 6)^2} \\ &= \sqrt{(-3)^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ d &= 5 \end{aligned}$$

Exercises

Use the distance formula to find the distance between each pair of points.

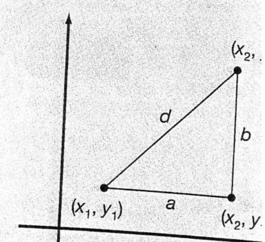
1. (8, −5) and (3, 7)
2. (0, 4) and (−4, 6)
3. (−3, −5) and (−6, −8)
4. (5, 6) and (−2, 6)
5. (−4, −4) and (4, 4)
6. (7, 0) and (−6, 4)

Exercises

1. 13
2. $2\sqrt{5}$
3. $3\sqrt{2}$
4. 7
5. $8\sqrt{2}$
6. $\sqrt{185}$

The Distance Formula

You may want to present tl
proof of the distance formu



Let segment *a* be horizontal
segment *b* be vertical. Since a
horizontal, its length is simply
The length of *b* is $y_2 - y_1$. Usi
Pythagorean theorem, $d^2 = a$
substitute for *a* and *b*.

$d^2 = (x_2 - x_1)^2 + (y_2 - y$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 -}$

Use Teaching Transparency T4
the distance formula.

11-8 Exercises

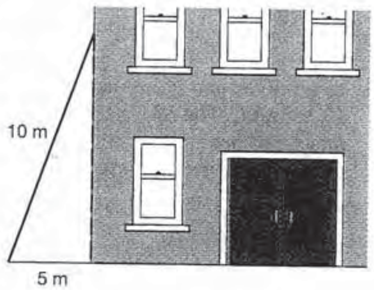
Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

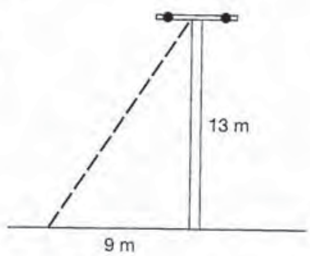
A

Solve. Round answers to nearest tenth.

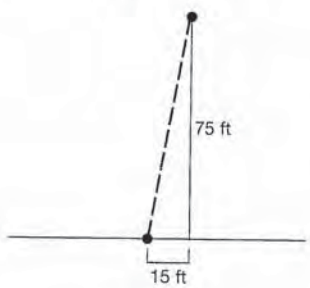
1. A 10-m ladder is leaning against a building. The bottom of the ladder is 5 m from the building. How high is the top of the ladder?



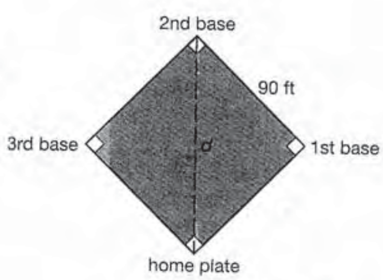
2. How long must a wire be to reach from the top of a 13-m telephone pole to a point on the ground 9 m from the foot of the pole?



3. What amount of wire is needed to connect the top of the antenna to the hook 15 ft from the base of the antenna, as shown at right?



4. The distance between consecutive bases in professional baseball is 90 ft. Find the distance from home plate to second base.



3. PRACTICE/ASSESS

Lesson Quiz

1. A support cable stretches fron of a 40-foot antenna tower to 30 feet out from the base of t tower. How long is the cable? 50 feet
2. A square gate is braced with a stretched across its diagonal. T length of the cable is $\sqrt{8}$ feet. wide is the gate? 2 feet

Assignment Guide

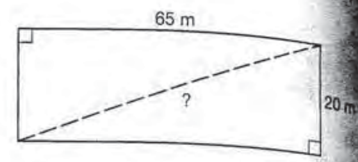
Core 1–18
Extension 19, 20

Use Mixed Review to maintain s

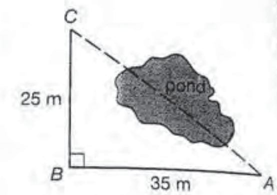
Exercises

1. ≈ 8.7 m
2. ≈ 15.8 m
3. ≈ 76.5 ft
4. ≈ 127.3 ft

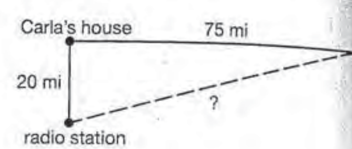
5. What is the distance across the garden shown at the right?



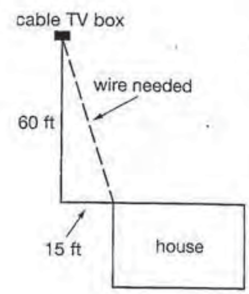
6. A surveyor had poles marked at points A, B, and C. The distances that could be measured are shown on the drawing. What is the approximate distance from A to C ?



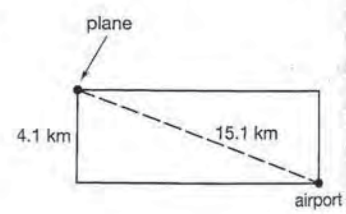
7. Carla Chew lives 20 miles due north of her favorite radio station. While driving due east from her house, she was able to keep the radio signal for about 75 miles. What is the broadcasting range of her favorite radio station?



8. A cable television company needed to wire from its box at the corner of a lot to a corner of a house. The owner knew the house was 15 ft from the side of the lot and 60 ft from the back of the lot. How much wire was needed to go from the box to the house?



9. An airplane is flying at an altitude of 4.1 km. The plane's slant distance from the runway is 15.1 km. How far must the plane travel to be directly above the runway?

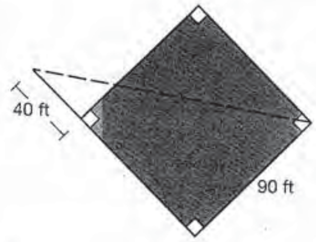


Exercises

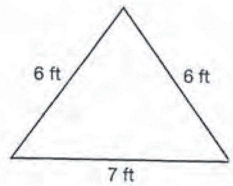
5. ≈ 68.0 m
6. ≈ 43.0 m
7. ≈ 77.6 mi
8. ≈ 61.8 ft
9. ≈ 14.5 km

B

10. Suppose an outfielder catches the ball on the third base line about 40 feet behind third base. About how far would the outfielder have to throw the ball to first base?

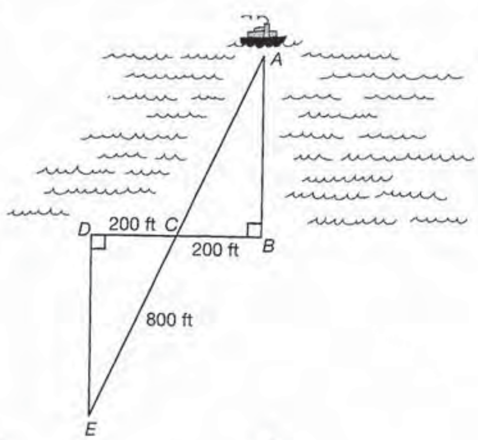


11. An A-frame tent is in the shape of an isosceles triangle. The base of the triangle is 7 ft and the two congruent sides are each 6 ft. What is the height of the tent?

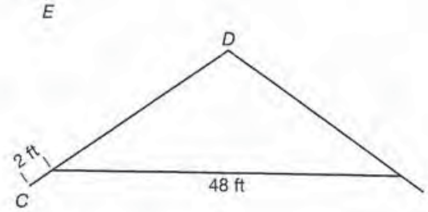


Suppose the tent in Exercise 11 is 9 ft long. Find the space inside (volume) of the tent.

12. How far out is the ship? Using geometry, Kala found triangle CDE to be *congruent* (same size and shape) to triangle CBA.



13. A 48-ft-wide building has a roof that rises 4 ft for each 12-foot horizontal change. If the roof has a 2-foot overhang, what is the length of $\overline{CD}$?



14. The diagonal of a square has length $8\sqrt{2}$ ft. Find the length of a side of the square.

15. Find the length of the diagonal of a rectangle whose length is 12 in. and whose width is 7 in.

16. One leg of a right triangle is 5 times the other leg. Find the longer leg if the hypotenuse is 60 ft.

Exercises

10. $50\sqrt{10} \approx 158.1$ ft
11. $\sqrt{23.75} \approx 4.9$ ft

Photo caption: about 153.5 ft^3

12. $200\sqrt{15} \approx 774.6$ ft
13. $8\sqrt{10} + 2 \approx 27.3$ ft
14. 8 ft
15. $\sqrt{193} \approx 13.9$ in.
16. 58.8 ft

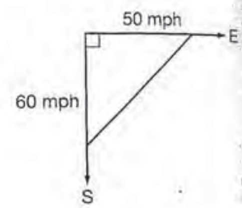
17. ***Critical Thinking*** In Exercise 13, suppose that the roof rises 3 feet for each 8-foot horizontal change, and that all other measurements remain the same. What is the length of $\overline{CD}$?

18. ***Mathematical Reasoning*** Suppose one of your classmates claims that the Pythagorean theorem is $x^2 + y^2 = z^2$. Another classmate responds, "It depends." Write a paragraph explaining why the second classmate is correct.

Challenge

19. Two major highways meet at a 90° angle. The known distances between cities are marked on the map at the right. An old road connects these cities. On the major highways, one can average 50 mi/h from point A to point C. On the old road, one can average only 30 mi/h. Which path would get you from A to C in the least time?

20. Two cars leave a service station at the same time. One car travels east at a speed of 50 mi/h, and the other travels south at a speed of 60 mi/h. After one-half hour, how far apart are they? When will they be 100 miles apart?



Mixed Review

Rationalize the denominator. **21.** $\sqrt{\frac{3}{5}}$ **22.** $\frac{\sqrt{20}}{\sqrt{5y}}$ **23.** $\frac{\sqrt{y}}{\sqrt{xy}}$ *11-5*

Multiply and simplify. **24.** $\sqrt{15}\sqrt{10}$ **25.** $\sqrt{xy}\sqrt{xyz}$ *11-4*

Add or subtract. **26.** $\sqrt{2} + 4\sqrt{2}$ **27.** $4\sqrt{12} - 2\sqrt{3}$

28. $6\sqrt{12} - 4\sqrt{3}$ **29.** $\sqrt{45} + \sqrt{20}$ **30.** $2\sqrt{8} + 6\sqrt{18}$

31. $\sqrt{24} - \sqrt{6}$ **32.** $\sqrt{18} - 5\sqrt{8} + \sqrt{72}$ *11-6*

Exercises

17. $3\sqrt{73} + 2 \approx 27.6$ ft
18. Answers may vary, but students should recognize that the square of the hypotenuse equals the sum of the squares of the legs, regardless of what variables are used to name them.
19. Old road 2.5 h, major highways 2.1 h; major highways
20. $\sqrt{1525} = 5\sqrt{61} \approx 39.05$ mi, 1.28 h

Mixed Review

21. $\frac{\sqrt{15}}{5}$
22. $\frac{2\sqrt{y}}{y}$
23. $\frac{\sqrt{x}}{x}$
24. $5\sqrt{6}$
25. $xy\sqrt{z}$
26. $5\sqrt{2}$
27. $6\sqrt{3}$
28. $8\sqrt{3}$
29. $5\sqrt{5}$
30. $22\sqrt{2}$
31. $\sqrt{6}$
32. $-\sqrt{2}$

Try This Approximate to the nearest meter.

f. A person can see 61 km to the horizon from the roof of a building. How high is the rooftop?

11-9 Exercises

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A

Solve.

1. $\sqrt{x} = 5$	**2.** $\sqrt{x} = 7$
3. $\sqrt{x} = 6.2$	**4.** $\sqrt{x} = 4.3$
5. $\sqrt{x + 3} = 20$	**6.** $\sqrt{x + 4} = 11$
7. $\sqrt{2x + 4} = 25$	**8.** $\sqrt{2x + 1} = 13$
9. $3 + \sqrt{x - 1} = 5$	**10.** $4 + \sqrt{y - 3} = 11$
11. $6 - 2\sqrt{3n} = 0$	**12.** $8 - 4\sqrt{5n} = 0$
13. $\sqrt{5x - 7} = \sqrt{x + 10}$	**14.** $\sqrt{4x - 5} = \sqrt{x + 9}$
15. $\sqrt{x} = -7$	**16.** $-\sqrt{x} = 5$
17. $\sqrt{2y + 6} = \sqrt{2y - 5}$	**18.** $2\sqrt{x - 2} = \sqrt{7 - x}$

Solve. Use the formula $V = 3.5\sqrt{h}$ for Exercises 19–22.

19. How far can you see to the horizon from an airplane at 9800 m?

20. How far can a sailor see to the horizon from the top of a 24-m mast?

21. A person can see about 350 km to the horizon from an airplane window. How high is the airplane?

22. A person can see about 100 km to the horizon from the top of a hill. How high is the hill?

The formula $r = 2\sqrt{5L}$ can be used to approximate the speed (r), in mi/h, of a car that has left a skid mark of length L, in feet.

23. How far will a car skid at 50 mi/h? at 70 mi/h?

24. How far will a car skid at 60 mi/h? at 100 mi/h?

B

Solve.

25. $\sqrt{5x^2 + 5} = 5$

26. $\sqrt{x} = -x$

27. Find a number such that twice its square root is 14.

28. Find a number such that the opposite of three times its square root is -33.

3. PRACTICE/ASSESS

LESSON QUIZ

1. Solve $\sqrt{x + 5} = 7$.
   $x = 44$
2. Solve using $V = 3.5\sqrt{h}$.
   A person can see 140 km to tl
   horizon from an airplane win
   How high is the airplane?
   The airplane is at an altitude
   1600 meters.

Assignment Guide

To provide flexible scheduling
lesson can be split into parts.

- Core 1–18, 25–28, 31, 32
  Extension 33–39
- Core 19–24, 29, 30
  Extension 40–42

Use Mixed Review to maintain

Try This

d. ≈ 313 km
e. ≈ 16 km
f. About 300 m

Exercises

1. 25
2. 49
3. 38.44
4. 18.49
5. 397
6. 117
7. 310.5
8. 84
9. 5
10. 52
11. 3
12. $\frac{4}{5}$
13. $\frac{17}{4}$
14. $\frac{14}{3}$
15. No value
16. No value
17. No value
18. 3
19. ≈ 346 km
20. ≈ 17 km
21. ≈ 10,000 m
22. ≈ 816 m

Note: $L = \frac{r^2}{20}$

23. 125 ft, 245 ft
24. 180 ft, 500 ft
25. 2 or −2
26. 0
27. 49
28. 121

The formula $T = 2\pi\sqrt{\frac{L}{32}}$ can be used to find the period (T), in seconds, of a pendulum of length L, in feet.

29. What is the length of a pendulum that has a period of 1.6 sec? Use 3.14 for π.

30. What is the length of a pendulum that has a period of 3 sec? Use 3.14 for π.

31. ***Critical Thinking*** Find a number such that the square root of 4 more than 5 times the number is 8.

32. ***Mathematical Reasoning*** Justify each step to prove the Principle of Squaring.

a. $a = b$ ____________

b. $a \cdot a = a \cdot b$ ____________

c. $a \cdot b = b \cdot b$ ____________

d. $a \cdot a = b \cdot b$ ____________

e. $a^2 = b^2$ ____________

Challenge

Solve.

33. $x - 1 = \sqrt{x + 5}$

34. $\sqrt{y^2 + 6} + y - 3 = 0$

35. $\sqrt{x - 5} + \sqrt{x} = 5$ (Use the principle of squaring twice.)

36. $\sqrt{3x + 1} = 1 + \sqrt{x + 4}$

37. $4 + \sqrt{10 - x} = 6 + \sqrt{4 - x}$

38. $x = (x - 2)\sqrt{x}$

39. Solve $A = \sqrt{1 + \frac{a^2}{b^2}}$ for b.

The formula $t = \sqrt{\frac{2s}{g}}$ gives the time in seconds for an object, initially at rest, to fall s feet.

40. Solve the formula for s.

41. If $g = 32.2$, find the distance an object falls in the first 5 seconds.

42. Find the distance an object falls in the first 10 seconds.

Mixed Review

Determine the replacements for the variable that give real numbers.

43. $\sqrt{x + 1}$ **44.** $\sqrt{x - 1}$ **45.** $\sqrt{5x + 1}$ **46.** $\sqrt{x^2 + 4}$ *11-1*

Simplify. **47.** $\sqrt{225}$ **48.** $\sqrt{m^2}$ **49.** $\sqrt{x^2 - 2x + 1}$

50. $\sqrt{(-9a)^2}$ **51.** $\sqrt{y^{13}}$ **52.** $\sqrt{9(y + 7)^4}$ **53.** $\sqrt{144m^5}$ *11-2*

Solve. **54.** $y^2 - \frac{8}{15}y + \frac{15}{225} = 0$ **55.** $x^2 + \frac{1}{12}x - \frac{12}{144} = 0$ *6-5*

Practice Multiple Choice

CALIFORNIA ALGEBRA STANDARDS

Choose the best answer.

1. Find the length of the hypotenuse of a right triangle if the legs are 5 and 13.

A $2\sqrt{97}$
B $\sqrt{194}$
C 12
D 9

2. Which statement is an example of deductive reasoning?

F If $\sqrt{x} + 10 = 19$, then $x = 81$.
G It has rained the last two days, so it will rain today.
H Both of the above.
J None of the above.

1. B; Algebra 2.0
2. F; Algebra 24.1

Exercises

Note: $L = 8\left(\frac{T}{\pi}\right)^2$

29. 2.1 ft
30. 7.3 ft
31. 12
32. a. Given
b. Multiplication Property of Equality
c. Multiplication Property of Equality
d. Transitive Property
e. Definition of square number
33. 4
34. $\frac{1}{2}$
35. 9
36. 5
37. $\frac{15}{4}$
38. 0 or 4 (1 is not a solution.)
39. $\pm\sqrt{\frac{a^2}{A^2 - 1}}$
40. $\frac{t^2g}{2}$
41. 402.5 ft
42. 1610 ft

Mixed Review

43. $x \geq -1$
44. $x \geq 1$
45. $x \geq -\frac{1}{5}$
46. Any real number
47. 15
48. $|m|$
49. $|x - 1|$
50. $9|a|$
51. $y^6\sqrt{y}$
52. $3(y + 7)^2$
53. $12m^2\sqrt{m}$
54. $\frac{1}{3}, \frac{1}{5}$
55. $-\frac{1}{3}, \frac{1}{4}$

11 Chapter 11 Wrap Up

11-1

The number c is a **square root** of a if $c^2 = a$. The square roots of 64 are 8 and -8. The **principal square root** of 64 is written $\sqrt{64} = 8$. The negative square root of 64 is written $-\sqrt{64} = -8$.

Simplify.

1. $\sqrt{36}$ **2.** $-\sqrt{81}$ **3.** $\sqrt{49}$ **4.** $-\sqrt{169}$

An **irrational number** cannot be named by fractional notation $\frac{a}{b}$. The rational numbers and irrational numbers make up the set of **real numbers.**

Identify each square root as rational or irrational.

5. $\sqrt{3}$ **6.** $\sqrt{36}$ **7.** $-\sqrt{12}$ **8.** $-\sqrt{4}$

11-2

In a radical expression, the expression written under the radical $\sqrt{x^2 + 5}$ is called the **radicand.** Radical expressions with negative radicands have no meaning in the real number system.

Determine the replacements for the variable so that the expression represents a real number.

9. $\sqrt{x + 7}$ **10.** $\sqrt{x - 10}$

Simplify.

11. $\sqrt{m^2}$ **12.** $\sqrt{49t^2}$ **13.** $\sqrt{p^2}$ **14.** $\sqrt{(x - 4)^2}$

11-3

For any nonnegative numbers, a and b, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. You can use this property to simplify radical expressions. A simplified radical expression has no factors under the radical sign that are perfect squares.

To find the square root of an even power such as x^{10}, take half of the exponent. If the exponent is odd, write the power as a product of the largest even power and x. Then simplify the even power.

Factor and simplify. Assume that all variables are nonnegative.

15. $-\sqrt{48}$ **16.** $\sqrt{x^2 - 14x + 49}$ **17.** $\sqrt{64x^2}$

18. $\sqrt{36x}$ **19.** $\sqrt{x^{12}}$ **20.** $\sqrt{y^5}$

21. $\sqrt{(x - 2)^4}$ **22.** $\sqrt{75y^{15}}$

23. $\sqrt{25x^9}$ **24.** $\sqrt{(y + 7)^{10}}$

Key Terms

conjugate (p. 506)
cube root (p. 494)
distance formula (p. 513)
division property for radicals (p. 498)
hypotenuse (p. 509)
irrational number (p. 483)
leg (p. 509)
principal square root (p. 482)
principle of squaring (p. 519)
product property for radicals (p. 491)
Pythagorean theorem (p. 509)
radical equation (p. 519)
radical expression (p. 487)
radical sign (p. 482)
radicand (p. 487)
rationalizing the denominator (p. 499)
real number (p. 483)
square root (p. 482)

CHAPTER 11 WRAP UP

1. 6
2. -9
3. 7
4. -13
5. Irrational
6. Rational
7. Irrational
8. Rational
9. $x \geq -7$
10. $x \geq 10$
11. $|m|$
12. $7|t|$
13. $|p|$
14. $|x - 4|$
15. $-4\sqrt{3}$
16. $x - 7$
17. $8x$
18. $6\sqrt{x}$
19. x^6
20. $y^2\sqrt{y}$
21. $(x - 2)^2$
22. $5y^7\sqrt{3y}$
23. $5x^4\sqrt{x}$
24. $(y + 7)^5$

Internet Activity On the Web

Look for extension problems for this chapter at the Prentice Hall Web site.
www.phschool.com

11-4

We can use the product property for radicals to multiply radicals. Sometimes we can simplify after multiplying. We can find perfect square factors and take their square roots.

Multiply.

25. $\sqrt{3} \cdot \sqrt{7}$

26. $\sqrt{a} \cdot \sqrt{t}$

27. $\sqrt{x-3} \cdot \sqrt{x+3}$

28. $\sqrt{2x} \cdot \sqrt{3y}$

29. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{7}}$

30. $\sqrt{3x} \cdot \sqrt{2x+1}$

Multiply and simplify.

31. $\sqrt{3} \cdot \sqrt{6}$

32. $\sqrt{2x^2} \cdot \sqrt{5x^5}$

33. $\sqrt{ab} \cdot \sqrt{bc}$

34. $\sqrt{5b} \cdot \sqrt{15b^3}$

11-5

For any nonnegative radicands A and B, $\frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}$.

A simplified expression may not have a radical in the denominator or a fractional radicand; you must remove the fraction from the radicand or the radical from the denominator by **rationalizing the denominator.**

Simplify.

35. $\sqrt{\frac{9}{16}}$

36. $\sqrt{\frac{1}{25}}$

37. $\sqrt{\frac{20}{45}}$

38. $\sqrt{\frac{9}{32}}$

Divide.

39. $\frac{\sqrt{48}}{\sqrt{3}}$

40. $\frac{\sqrt{45x^4}}{\sqrt{9}}$

41. $\frac{\sqrt{100x^3}}{\sqrt{25x^3}}$

42. $\frac{\sqrt{80y^4}}{\sqrt{5y^4}}$

Rationalize the denominator.

43. $\frac{\sqrt{3}}{\sqrt{5}}$

44. $\frac{5}{\sqrt{3}}$

45. $\frac{\sqrt{8}}{\sqrt{x}}$

46. $\frac{\sqrt{64a^3}}{\sqrt{6}}$

47. $\sqrt{\frac{1}{8}}$

48. $\sqrt{\frac{5}{y}}$

CHAPTER 11 WRAP UP

25. $\sqrt{21}$
26. $\sqrt{at}$
27. $\sqrt{x^2-9}$
28. $\sqrt{6xy}$
29. $\sqrt{\frac{15}{28}}$
30. $\sqrt{6x^2+3x}$
31. $3\sqrt{2}$
32. $x^3\sqrt{10x}$
33. $b\sqrt{ac}$
34. $5b^2\sqrt{3}$
35. $\frac{3}{4}$
36. $\frac{1}{5}$
37. $\frac{2}{3}$
38. $\frac{3\sqrt{2}}{8}$
39. 4
40. $x^2\sqrt{5}$
41. 2
42. 4
43. $\frac{\sqrt{15}}{5}$
44. $\frac{5\sqrt{3}}{3}$
45. $\frac{2\sqrt{2x}}{x}$
46. $\frac{4a\sqrt{6a}}{3}$
47. $\frac{\sqrt{2}}{4}$
48. $\frac{\sqrt{5y}}{y}$

11-6

To add or subtract real numbers with the same radicand, use the distributive property. You may need to simplify the radicals first.

Add or subtract.

49. $10\sqrt{5} + 3\sqrt{5}$

50. $\sqrt{80} - \sqrt{45}$

51. $\sqrt{x} + \sqrt{9x}$

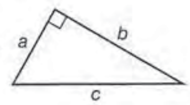
52. $3\sqrt{2} - 5\sqrt{\frac{1}{2}}$

53. $\sqrt{9x + 9} + \sqrt{x + 1}$

54. $\sqrt{12x^2} + \sqrt{3x^2}$

11-7

The **Pythagorean theorem,** $c^2 = a^2 + b^2$, can be used to find the hypotenuse (c) or the legs (a and b) of a right triangle.



In a right triangle, find the length of the side not given.

55. $a = 15, b = 20$

56. $c = 5\sqrt{2}, b = 5$

57. $a = 6, c = 10$

58. $a = \sqrt{2}, b = \sqrt{5}$

59. $c = 18, b = 14$

60. $b = 6, a = 6\sqrt{3}$

11-8

Use the Pythagorean theorem and the Problem-Solving Guidelines to help you solve problems with right triangles.

Solve.

61. An 18-ft ladder leans against a house, reaching a point 14 ft above the ground. How far is the foot of the ladder from the bottom of the house?

62. How long must a wire be to reach from the top of a 12-ft pole to a point on the ground 8 ft from the pole?

11-9

To solve an equation with radicals, square both sides of the equation. You must always check for **extraneous solutions,** which are not solutions to the original equation.

Solve.

63. $\sqrt{x - 3} = 7$

64. $\sqrt{3x} - 8 = 13$

Solve. Use the formula $V = 3.5\sqrt{h}$.

65. A person can see 75.6 km to the horizon from the top of a mountain. How high is the mountain?

CHAPTER 11 WRAP UP

49. $13\sqrt{5}$
50. $\sqrt{5}$
51. $4\sqrt{x}$
52. $\frac{\sqrt{2}}{2}$
53. $4\sqrt{x + 1}$
54. $3x\sqrt{3}$
55. $c = 25$
56. $a = 5$
57. $b = 8$
58. $c = \sqrt{7}$
59. $a = 8\sqrt{2}$
60. $c = 12$
61. $\sqrt{128} \approx 11.3$ ft
62. $\sqrt{208} \approx 14.4$ ft
63. 52
64. 147
65. 466.56 m

ent Item Analysis

Lesson
11-1
11-2
11-3
11-4
11-5
11-6
11-7
11-8
11-9

11 Chapter Assessment

Simplify.

1. $\sqrt{64}$
2. $-\sqrt{25}$

Identify each number as rational or irrational.

3. $\sqrt{16}$
4. $-\sqrt{10}$

Simplify. The variables represent any real number.

5. $\sqrt{a^2}$
6. $\sqrt{36y^2}$
7. $\sqrt{(y+2)^2}$

Simplify. Assume that all variables are nonnegative.

8. $-\sqrt{40}$
9. $\sqrt{27}$
10. $\sqrt{25x - 25}$
11. $\sqrt{x^6}$
12. $\sqrt{y^9}$
13. $\sqrt{(y+2)^4}$

Multiply.

14. $\sqrt{3} \cdot \sqrt{11}$
15. $\sqrt{3x} \cdot \sqrt{y}$
16. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{5}{7}}$

Multiply and simplify. Assume that all variables are nonnegative.

17. $\sqrt{5} \cdot \sqrt{10}$
18. $\sqrt{3ab} \cdot \sqrt{6ab^3}$
19. $\sqrt{xy} \cdot \sqrt{y^2}$

Simplify. Assume that all variables are nonnegative.

20. $\sqrt{\frac{27}{12}}$
21. $\sqrt{\frac{144}{a^2}}$
22. $\sqrt{\frac{1}{36}}$

Divide. Assume that all radicands are nonnegative.

23. $\frac{\sqrt{36}}{\sqrt{12}}$
24. $\frac{\sqrt{75x^4}}{\sqrt{3}}$
25. $\frac{\sqrt{96y^3}}{\sqrt{16y}}$

Rationalize the denominator.

26. $\frac{\sqrt{2x}}{\sqrt{y}}$
27. $\frac{7}{\sqrt{2}}$
28. $\sqrt{\frac{2}{5}}$

Add or subtract.

29. $3\sqrt{18} - 5\sqrt{18}$
30. $\sqrt{5} + \sqrt{\frac{1}{5}}$
31. $\sqrt{20x^2} + \sqrt{45x^2}$
32. In a right triangle, the length of the hypotenuse is 91 and the length of one of the legs is 84. Find the length of the missing leg.
33. A slow pitch softball diamond is a square 65 ft on a side. How far is it from home to second base?

Solve.

34. $\sqrt{3x} + 2 = 14$
35. $\sqrt{y - 2} + 3 = 9$
36. Use $r = 2\sqrt{5d}$. A car's rate (r) is 55 mi/h. How far (d) will it skid?

CHAPTER 11 ASSESSMENT

1. 8
2. -5
3. Rational
4. Irrational
5. $|a|$
6. $6|y|$
7. $|y + 2|$
8. $-2\sqrt{10}$
9. $3\sqrt{3}$
10. $5\sqrt{x - 1}$
11. x^3
12. $y^4\sqrt{y}$
13. $(y + 2)^2$
14. $\sqrt{33}$
15. $\sqrt{3xy}$
16. $\sqrt{\frac{10}{21}}$
17. $5\sqrt{2}$
18. $3ab^2\sqrt{2}$
19. $y\sqrt{xz}$
20. $\frac{3}{2}$
21. $\frac{12}{a}$
22. $\frac{1}{6}$
23. $\sqrt{3}$
24. $5x^2$
25. $y\sqrt{6}$
26. $\frac{\sqrt{2xy}}{y}$
27. $\frac{7\sqrt{2}}{2}$
28. $\frac{\sqrt{10}}{5}$
29. $-6\sqrt{2}$
30. $\frac{6}{5}\sqrt{5}$
31. $5x\sqrt{5}$
32. 35
33. $\approx$ 92 ft
34. 48
35. 38
36. 151.25 ft

1–11 Cumulative Review

1-4 Evaluate.

1. $x - (x - 1) + (x^2 + 1)$ for $x = 10$
2. $xy + (xy)^2 + x + y$ for $x = 5$ and $y = 2$

1-5 Factor.

3. $121a + 88b$
4. $36x + 24y + 12z$

1-6 Write as an algebraic expression.

5. the product of x and 4 less than y divided by the difference of x and 8 times y
6. 19 more than the fifth power of a

2-3, 2-4 Simplify.

7. $-12.1 + 100.6 - 18.5$
8. $\frac{11}{9} - \frac{4}{18} + \frac{1}{6} - \frac{2}{3} + \frac{1}{2}$

2-5, 2-6 Multiply or divide.

9. $6(-12)$
10. $(-2.3)(-4.4)$
11. $\frac{-48}{-6}$
12. $\frac{18.6}{-6.2}$

2-7 Factor.

13. $-21x - 28w$
14. $144a^2 - 60b^2$

2-8 Simplify.

15. $-7(x - 1) + 2x$
16. $8[4 - (6x - 5)]$

3-4 Solve.

17. During the library marathon, Luis read twice as many books as Roger, but only half as many as Huong. If the three boys read 21 books, how many did Roger read?

$L + 2L + \frac{L}{2} = 21$

3-5 Solve.

18. $5(a + 3) = 8a + 18$
19. $-3(m + 2) - 4 = -12 - (-2 + m)$

CHAPTERS 1–11 CUMULATIVE REVIEW

1. 102
2. 117
3. $11(11a + 8b)$
4. $12(3x + 2y + z)$
5. $\frac{x(y - 4)}{x - 8y}$
6. $a^5 + 19$
7. 70
8. 1
9. -72
10. 10.12
11. 8
12. -3
13. $-7(3x + 4w)$
14. $12(12a^2 - 5b^2)$
15. $-5x + 7$
16. $72 - 48x$
17. 3 books
18. -1
19. 0

3-10 Translate to an equation and solve.

20. What percent of 1 is $\frac{1}{2}$?

21. 50 is 20% of what number?

22. 250% of what number is 6.25?

3-11 Translate to an equation and solve.

23. The ratio of apples to oranges sold in a local supermarket is 5 to 7. If 1320 pieces of fruit were sold in one week, how many were oranges?

4-4 Solve.

24. $8x - 2 \geq 7x + 5$

25. $-4x \geq 24$

26. $-3x < 30 + 2x$

27. $x + 3 > 6(x - 4) + 7$

4-5 Solve.

28. Amy received grades of 85, 87, 88, and 92 on her math tests. What must her grade be on her next test if her average is to be at least 90?

5-1, 5-2 Simplify.

29. $(a^2b^4)(ab^5)$

30. $x^4 \cdot x^6 \cdot x$

31. $\frac{a^8b^4}{a^5b^3}$

32. a^0

33. 7^{-4}

34. $(3^3)^2$

35. $(-3x^5y^2)^3$

36. $\left(\frac{y^4}{2}\right)^3$

5-4 Write using scientific notation.

37. 346,000

38. 0.0000628

5-5 Collect like terms.

39. $10a^2 + 6a - 8a^2 + 3a - 2a^2 - 8a$

40. $(8m + 6n) - (12n + 7m) + 2(4m - 11n)$

5-9, 5-10 Multiply.

41. $(4 + 2b + c)(c - 1)$

42. $(2.5x - 0.3y)^2$

43. $(5x - 6y)(x + 2y)$

44. $(a + 2c + 1)(a + 2c - 1)$

45. $(h - 4k)^2$

46. $(2x + 7y)^2$

47. $(x^2 + 3)(x^2 - 3)$

48. $(4x + 2)(3x - 1)$

CHAPTERS 1–11 CUMULATIVE REVIEW

20. 50%
21. 250
22. 2.5
23. 770 oranges
24. $x \geq 7$
25. $x \leq -6$
26. $x > -6$
27. $x < 4$
28. 98
29. a^3b^9
30. x^{11}
31. a^3b
32. 1
33. $\frac{1}{7^4}$
34. 3^6
35. $-27x^{15}y^6$
36. $\frac{y^{12}}{8}$
37. 3.46×10^5
38. 6.28×10^{-5}
39. a
40. $9m - 28n$
41. $3c + 2bc + c^2 - 4 - 2b$
42. $6.25x^2 - 1.5xy + 0.09y^2$
43. $5x^2 + 4xy - 12y^2$
44. $a^2 + 4ac + 4c^2 - 1$
45. $h^2 - 8hk + 16k^2$
46. $4x^2 + 28xy + 49y^2$
47. $x^4 - 9$
48. $12x^2 + 2x - 2$

6-1 to 6-7 Factor.

49. $x^3y^3 + x^2y^2 - 4xy$
50. $a^8 + a^7 - a^6$
51. $4 - 9m^4$
52. $9x^2 - 9b^2$
53. $x^2 - 4x - 12$
54. $s^2 - 16s + 15$
55. $t^3 - 5t^2 + 6t$
56. $7x^2 - 6x - 1$
57. $20y^2 + 19y + 3$
58. $6y^2 + 9y - 15$

6-8 Solve.

59. $x^2 - 3x - 10 = 0$
60. $y(3y - 2) = 0$

7-5, 7-6 Write an equation for the line that satisfies each condition.

61. slope is 5, passes through the origin
62. passes through (0, 3) and has x-intercept 6
63. slope is -1, y-intercept is -12
64. x-intercept is 7, y-intercept is -6
65. contains the points $(-1, 2)$ and $(2, 11)$

8-2, 8-3 Solve each system.

66. $6x + 3y = -6$
$-2x + 5y = 14$

67. $2x + 3y = -3$
$y = 2x - 9$

8-4 to 8-6

68. Find two numbers such that their sum is 337 and their difference is 43.
69. The units digit of a certain number is one greater than twice the tens digit. If the digits are reversed, the new number is 36 more than the original number. Find the original number.

9-1

Let $A = \{0, 2, 4, 6, 8, 10, 12\}$ and $B = \{0, -2, -4, -6, -8, -10, -12\}$. Find the following.

70. $A \cup B$
71. $A \cap B$

9-3 Solve.

72. $|2x + 4| = 12$
73. $|2x - 5| = 7$

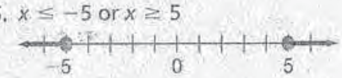
9-4 Solve and graph.

74. $|3y| < 12$
75. $|4x| \geq 20$

CHAPTERS 1–11 CUMULATIVE REVIEW

49. $xy(x^2y^2 + xy - 4)$
50. $a^6(a^2 + a - 1)$
51. $(2 - 3m^2)(2 + 3m^2)$
52. $9(x - b)(x + b)$
53. $(x - 6)(x + 2)$
54. $(s - 1)(s - 15)$
55. $t(t - 2)(t - 3)$
56. $(7x + 1)(x - 1)$
57. $(4y + 3)(5y + 1)$
58. $3(y - 1)(2y + 5)$
59. $-2, 5$
60. $0, \frac{2}{3}$
61. $y = 5x$
62. $y = -\frac{1}{2}x + 3$
63. $y = -x - 12$
64. $y = \frac{6}{7}x - 6$
65. $y = 3x + 5$
66. $(-2, 2)$
67. $(3, -3)$
68. 147, 190
69. 37
70. $\{-12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12\}$
71. $\{0\}$
72. $4, -8$
73. $6, -1$
74. $-4 < y < 4$

75. $x \leq -5$ or $x \geq 5$



10-2 Multiply. Simplify the product.

76. $\frac{3y}{y+1} \cdot \frac{y+1}{2}$

77. $\frac{-4}{3y+3} \cdot \frac{y+1}{2}$

10-3 Divide. Simplify the quotient.

78. $\frac{2x-6}{5} \div \frac{x^2-9}{15}$

79. $\frac{x^2y + x^3y}{x} \div \frac{1+x}{x^2y}$

10-4, 10-5 Add or subtract. Simplify.

80. $\frac{4}{2x+8} + \frac{6}{x+4}$

81. $\frac{3y}{y+1} - \frac{2y+6}{y^2-1}$

82. $\frac{y^2}{x-2} + \frac{5}{x-2} - \frac{3x}{x-2}$

10-6 Solve.

83. $\frac{3}{x-3} - \frac{2}{x+3} = \frac{5}{x^2-9}$

84. $\frac{12}{y} - \frac{12}{y+1} = 1$

10-7

85. It takes two computers working together 10 hours to solve problems about a tail design for an airplane. One computer working alone can do the job in 16 hours. How long does it take the other computer to do the job alone?

10-9 Divide.

86. $x^2 + 5x - 28$ by $x + 8$

10-10 Simplify.

87. $\frac{\frac{1}{x} + 3}{\frac{1}{x} - 2}$

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76. $\frac{3y}{2}$
77. $-\frac{2}{3}$
78. $\frac{6}{x+3}$
79. x^3y^2
80. $\frac{8}{x+4}$
81. $\frac{3y^2-5y-6}{(y+1)(y-1)}$
82. $\frac{y^2+5-3x}{x-2}$
83. -10
84. $-4, 3$
85. $26\frac{2}{3}$ h or 26 h 40 min
86. $x - 3 - \frac{4}{x+8}$
87. $\frac{1+3x}{1-2x}$

11-1, 11-2 Simplify.

88. $\sqrt{49}$
89. $-\sqrt{121}$
90. $\sqrt{64p^2}$
91. $\sqrt{(-7c)^2}$
92. $\sqrt{(x-2)^2}$
93. $\sqrt{x^2-6x+9}$

11-3, 11-4 Simplify.

94. $-\sqrt{56}$
95. $\sqrt{x^8}$
96. $\sqrt{36b^5}$
97. $\sqrt{20x^4y^5}$

Multiply. Simplify where possible.

98. $\sqrt{6}\cdot\sqrt{8}$
99. $\sqrt{6a}\,\sqrt{3a^2b^2}$
100. $\sqrt{2b}\cdot\sqrt{5a+3b}$
101. $\sqrt{x+4y}\cdot\sqrt{2x-5y}$

11-5 Divide. Simplify where possible.

102. $\sqrt{\frac{1}{81}}$
103. $\frac{\sqrt{200x^3}}{\sqrt{25x}}$
104. $\frac{6}{\sqrt{2}}$
105. $\sqrt{\frac{1}{9}}$

11-6 Add or subtract.

106. $6\sqrt{2}-8\sqrt{2}$
107. $\sqrt{200}-\sqrt{8}$
108. $\sqrt{7}+\sqrt{\frac{1}{7}}$
109. $\sqrt{\frac{3}{4}}-\sqrt{\frac{4}{3}}$

11-7

In a right triangle, find the length of the side not given.

110. $c = 15, a = 9$
111. $a = 16, b = 30$

11-8

112. Larry wants to fit a circular piece of glass, 86 in. in diameter, through a doorway measuring 30 in. by 80 in. Will the glass fit through the doorway? What is the maximum diameter that could fit through the doorway?

11-9 Solve.

113. $\sqrt{x-5}=3$
114. $\sqrt{2y-3}=\sqrt{y+6}$

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88. 7
89. -11
90. $8|p|$
91. $7|c|$
92. $|x-2|$
93. $|x-3|$
94. $-2\sqrt{14}$
95. x^4
96. $6b^2\sqrt{b}$
97. $2x^2y^2\sqrt{5y}$
98. $4\sqrt{3}$
99. $3|ab|\sqrt{2a}$
100. $\sqrt{10ab+6b^2}$
101. $\sqrt{2x^2+3xy-20y^2}$
102. $\frac{1}{9}$
103. $2|x|\sqrt{2}$
104. $3\sqrt{2}$
105. $\frac{1}{3}$
106. $-2\sqrt{2}$
107. $8\sqrt{2}$
108. $\frac{8\sqrt{7}}{7}$
109. $\frac{-\sqrt{3}}{6}$
110. $b = 12$
111. $c = 34$
112. No; about 85 in.
113. 14
114. 9