

wish to preteach some of the key
id in this chapter. Particularly for
anguage Learners (ELL),
g the vocabulary before the
r lesson begins gives students a
t into understanding the new
Writing new words on poster
inting to the words as you say
n displaying the poster for a
time is a useful technique.

- mmetry (p. 553)
- l variation (p. 566)
- of variation (p. 557)
- iation (p. 557)
- p. 536)
- (p. 537)
- values (p. 538)
- . 537)
- ariation (p. 561)
- ation (p. 565)
- iction (p. 547)
- p. 537)
- (p. 552)
- : function (p. 552)
- 536)
- p. 536)
- . 553)

Skills & Concepts you
need for ch. 12

$\frac{10}{3}$ 8.1050
20
-31
21
360
2A
 $b_1 + b_2$
 $\frac{c}{2a}$

b_2

CHAPTER 12

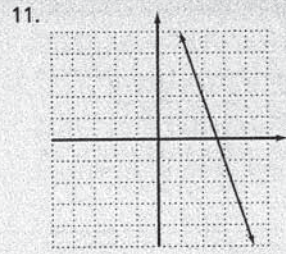
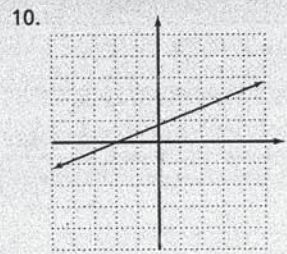
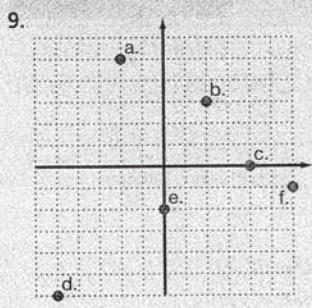
Skills & Concepts You Need for Chapter 12

What You'll Learn in Chapter 12

- How to recognize relations and functions and their graphs
- How to graph and use linear and quadratic functions
- How to use direct, inverse, joint, and combined variation

- 1-1, 1-3** Evaluate for $x = 5$ and $y = -3$.
1. $-\frac{2x}{y}$
 2. $x^2 + 1$
 3. $y^3 - 4$
- 1-9** Evaluate.
4. $A = \frac{1}{2}bh$ for $b = 7$ and $h = 6$.
 5. $I = Prt$ for $P = 1500$, $r = 0.12$, and $t = 2$.
- 3-7**
6. Solve $A = \frac{1}{2}h(b_1 + b_2)$ for h .
 7. Solve $C = 2\pi r$ for r .
- 3-9**
8. The winner of an election won by a vote of 5 to 3, getting 1750 votes. How many votes did the loser get?
- 7-1**
9. Plot these points on a coordinate plane.

a. $(-2, 5)$	b. $(2, 3)$	c. $(4, 0)$
d. $(-5, -6)$	e. $(0, -3)$	f. $(6, -1)$
- 7-3** Graph each equation.
10. $5y - 4 = 2x$
 11. $3x + y = 8$



Try This Find the indicated outputs.

- c. $f(x) = x + 3$; find $f(5)$, $f(-8)$, and $f(-2)$.
 d. $G(x) = 3x - x^2$; find $G(0)$, $G(-2)$, and $G(1)$.
 e. $f(y) = 8y^2 + 3$; find $f(-1)$, $f(2)$ and $f(\frac{1}{2})$.
 f. $p(x) = 2x^2 + x - 1$; find $p(0)$, $p(-2)$, and $p(3)$.

12-1 Exercises

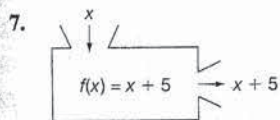
A Determine whether each relation is a function.

1. Domain Range 2. Domain Range 3. Domain Range
- | | | |
|--------|--------|--------|
| 2 → 9 | 5 → 3 | -5 → 1 |
| 5 → 8 | -3 → 7 | 5 → 1 |
| 19 → 8 | 7 → 7 | 8 → 1 |
| | -7 → 7 | |

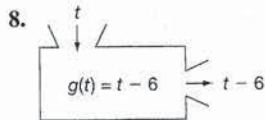
Find the domain and range of each relation. Is the relation a function?

4. $\{(6, -6), (7, -7), (3, -3), (4, -4)\}$
 5. $\{(a, b), (a, c), (d, e)\}$
 6. $\{(4, 1), (7, 1), (-2, 4), (1, 1)\}$

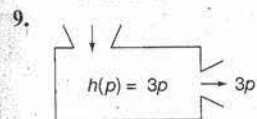
Find the indicated output for each function machine.



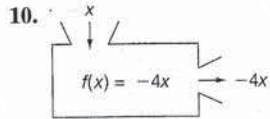
Find $f(3)$, $f(7)$, and $f(-9)$.



Find $g(0)$, $g(6)$, and $g(18)$.



Find $h(-2)$, $h(5)$, and $h(24)$.



Find $f(6)$, $f(-\frac{1}{2})$, and $f(20)$.

Find the indicated outputs for these functions.

11. $g(s) = 2s + 4$; find $g(1)$, $g(-7)$, and $g(6)$.
 12. $h(x) = 19$; find $h(4)$, $h(-6)$, and $h(12)$.
 13. $F(x) = 2x^2 - 3x + 2$; find $F(0)$, $F(-1)$, and $F(2)$.
 14. $P(x) = 3x^2 - 2x + 5$; find $P(0)$, $P(-2)$, and $P(3)$.

Try This

- c. 8, -5, 1
 d. 0, -10, 2
 e. 11, 35, 5
 f. -1, 5, 20

Exercises

1. Yes
 2. No
 3. Yes
 4. Domain: $\{3, 4, 6, 7\}$,
 Range: $\{-7, -6, -4, -3\}$; yes
 5. Domain: $\{a, d\}$,
 Range: $\{b, c, e\}$; no

6. Domain: $\{-2, 1, 4, 7\}$,
 Range: $\{1, 4\}$; yes
 7. 8, 12, -4
 8. -6, 0, 12
 9. -6, 15, 72
 10. -24, 2, -80
 11. 6, -10, 16
 12. 19, 19, 19
 13. 2, 7, 4
 14. 5, 21, 26



**Extra Help
 On the Web**

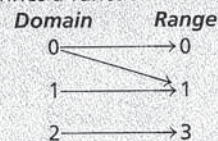
Look for worked-out examples at the Prentice Hall Web site.

www.phschool.com

3. PRACTICE/ASSESS

LESSON QUIZ

1. Determine whether the following defines a function.



No

2. Find the indicated outputs.
 $f(x) = 3x - 1$; find $f(0)$, $f(-1)$ and $f(5)$.
 $f(0) = -1$
 $f(-1) = -4$
 $f(5) = 14$

Assignment Guide

To provide flexible scheduling, lesson can be split into parts.

- ▼ Core 1-6
 Extension 32, 33
- ▼ Core 7-31
 Extension 34-40

Use Mixed Review to maintain:

Find the indicated outputs for these functions.

15. $h(x) = |x|$; find $h(-4)$, $h(4)$, and $h(-3)$.
16. $f(t) = |t| + 1$; find $f(-5)$, $f(0)$, and $f(-9)$.
17. $f(x) = |x| - 2$; find $f(3)$, $f(93)$, and $f(-100)$.
18. $g(t) = t^3 + 3$; find $g(1)$, $g(-5)$, and $g(0)$.
19. $h(x) = x^4 - 3$; find $h(0)$, $h(-1)$, and $h(3)$.
20. $f(m) = 3m^2 - 5$; find $f(4)$, $f(-3)$, and $f(6)$.

B

The cost of replacing a defective tire is a function of the tread depth. The chart below describes one such function. Refer to it for Exercises 21–24.

Tread Depth in Millimeters									
	9	8	7	6	5	4	3	2	1
% charged	0%	20%	30%	40%	55%	70%	80%	90%	100%

21. Find the cost of replacing a tire whose regular price is \$64.50 and whose tread depth is 4 mm.
22. Find the cost of replacing a tire whose regular price is \$78.50 and whose tread depth is 7 mm.
23. Find the cost of replacing a tire whose regular price is \$67.80 and whose tread depth is 3 mm.
24. Find the cost of replacing a tire whose regular price is \$72.40 and whose tread depth is 5 mm.
25. The function $P(d) = 1 + \frac{d}{33}$ gives the pressure of salt water in atmospheres as a function of d , the depth in feet. Find the pressure at 20 feet, 30 feet, and 100 feet.
26. The function $R(t) = 33\frac{1}{3}t$ gives the number of revolutions of a $33\frac{1}{3}$ RPM record as a function of t , the time in minutes it is on the turntable. Find the number of revolutions at 5 minutes, 20 minutes, and 25 minutes.
27. The function $T(d) = 10d + 20$ gives the temperature in degrees Celsius inside the earth as a function of d , the depth in kilometers. Find the temperature at 5 km, 20 km, and 1000 km.
28. The function $W(d) = 0.112d$ gives the depth of water in centimeters as a function of d , the depth of snow in centimeters. Find the depth of water that results from these depths of snow: 16 cm, 25 cm, and 100 cm.

Find the range of each function for the given domain.

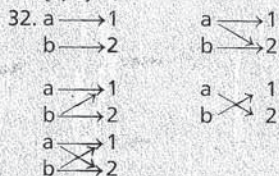
29. $f(x) = 3x + 5$ when the domain is the whole numbers less than 4.
30. $g(t) = t^2 - 5$ when the domain is the integers between -4 and 2 .
31. $h(x) = |x| - x$ when the domain is the integers between -2 and 20 .

540 Chapter 12 Relations and Functions

Exercises

15. 4, 4, 3
16. 6, 1, 10
17. 1, 91, 98
18. 4, -122 , 3
19. -3 , -2 , 78
20. 43, 22, 103
21. \$45.15
22. \$23.55
23. \$54.24
24. \$39.82
25. $1\frac{20}{33}$, $1\frac{30}{33}$, $4\frac{1}{33}$
26. $166\frac{2}{3}$, $666\frac{2}{3}$, $833\frac{1}{3}$

27. 70°C, 220°C, 10,020°C
28. 1.792 cm, 2.8 cm, 11.2 cm
29. {5, 8, 11, 14}
30. $\{-5, -4, -1, 4\}$
31. $\{0, 2\}$



33. a. Always; every domain element is assigned to exactly one range element.

- b. Sometimes; a domain element could be assigned to more than one range element.
- c. Never; at least one domain element must be assigned to two range elements.

34. 40
35. 100
36. -186
37. 880
38. -48
39. -64
40. $f(x) = \frac{15}{4}x - \frac{13}{4}$

32. **Critical Thinking** List all of the different relations between the elements in $\{a, b\}$ and $\{1, 2\}$.
33. **Mathematical Reasoning** Suppose that a relation has exactly n elements in its domain. For each of the following conditions, is the relation *sometimes*, *always*, or *never* a function? Justify your answers.
- The relation has exactly one element in its range.
 - The relation has exactly n elements in its range.
 - The relation has exactly $n + 1$ elements in its range.

Challenge

Use the functions $f(x) = 3x$ and $g(x) = -4x^2$ to find the following.

- $f(8) - g(2)$
- $2f(1) + 3g(4)$
- $f[g(-2)]$
- If $f(-1) = -7$ and $f(3) = 8$, find a linear equation for $f(x)$.
- $f(0) - g(-5)$
- $g(-3) \cdot f(-8) + 16$
- $g[g(-1)]$

Mixed Review

Solve.

41. $x - y = -7$	42. $2x + 3y = 11$	43. $2y + 5x = 4$
$x + y = 5$	$5y - x = 1$	$4x = 13 - 3y$
		8-1, 8-2, 8-3

Multiply.

44. $\sqrt{x}\sqrt{y}$	45. $\sqrt{y}\sqrt{y-2}$	46. $\sqrt{a}\sqrt{a+3}$
47. $\sqrt{m+n}\sqrt{m-n}$	48. $\sqrt{a+3}\sqrt{2a+1}$	
49. $\sqrt{x-4} \cdot \sqrt{3x+2}$	11-4	

Write an equation for the line that contains the given pair of points.

- | | |
|------------------------------|--|
| 50. $(-2, 2)$ and $(3, 7)$ | 51. $(5, 1)$ and $(-7, 1)$ |
| 52. $(6, -1)$ and $(-2, -5)$ | 53. $(3, 3)$ and $(3, -1)$ 7-6 |
| Simplify. | 54. $\sqrt{18}$ 55. $\sqrt{144m^2}$ 56. $\sqrt{20c^2}$ 57. $\sqrt{32a^3}$ 11-2, 11-3 |

Operations with Functions

We can perform arithmetic operations on functions.

EXAMPLE Suppose $f(x) = 3x$ and $g(x) = 2x + 1$. Find $f(x) + g(x)$ and $f(x) \cdot g(x)$.

$$f(x) + g(x) = 3x + (2x + 1) = 5x + 1$$

$$f(x) \cdot g(x) = 3x(2x + 1) = 6x^2 + 3x$$

Suppose $f(x) = 5x$, $g(x) = \frac{1}{x}$, and $h(x) = \frac{2}{(x-3)}$. Find the following.

- $f(x) - g(x)$
- $f(x) \cdot g(x)$
- $g(x) + h(x)$
- $g(x) \cdot h(x)$
- $g(x) \div h(x)$
- $f(x) \cdot h(x)$

12-1 Relations and Functions 541



California Topic:
CA 13.0: Add, subtract, multiply, and divide rational functions.

Mixed Review

- $(-1, 6)$
- $(4, 1)$
- $(-2, 7)$
- \sqrt{xy}
- $\sqrt{y^2 - 2y}$
- $\sqrt{a^2 + 3a}$
- $\sqrt{m^2 - n^2}$
- $\sqrt{2a^2 + 7a + 3}$
- $\sqrt{3x^2 - 10x - 8}$
- $y = x + 4$
- $y = 1$

- $y = \frac{1}{2}x - 4$
- $x = 3$
- $3\sqrt{2}$
- $12|m|$
- $2|c|\sqrt{5}$
- $4|a|\sqrt{2a}$

Operations with Functions

- $\frac{5x^2 - 1}{x}$
- 5 for $x \neq 0$
- $\frac{3x - 3}{x(x - 3)}$
- $\frac{2}{x(x - 3)}$
- $\frac{x - 3}{2x}$
- $\frac{10x}{x - 3}$

Finding the Domain of a Function

Use Teaching Transparency T50 as needed for the remaining classes to illustrate different types of functions.

Keyboard Examples

Find the domain of $f(x) = \frac{1}{x+3}$.
 $f(x)$ is not defined when $x + 3 = 0$,
 so, $x \neq -3$.
 The domain is $\{x \mid x \neq -3\}$.

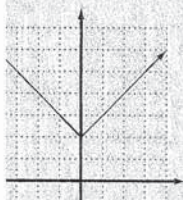
Find the domain of $g(x) = \sqrt{x+1}$.
 $g(x)$ is not defined when $x + 1 < 0$,
 so, $x + 1 \geq 0$, which occurs when
 $x \geq -1$.
 The domain is $\{x \mid x \geq -1\}$.

Use Teaching Transparency T51 for more examples.

Graphing Functions

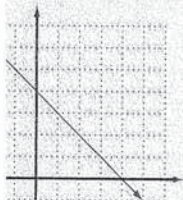
Keyboard Example

Graph $y = |x| + 2$. State the domain and range.



The domain is all real numbers. The range is $\{y \mid y \geq 2\}$.

Use this page 543.



All real numbers

Quick Review

To review set-builder notation, see Lesson 9-1.

PART 2 Finding the Domain of a Function

When a function is defined by an equation and the domain of the function is not stated, we assume that the domain is all real numbers for which the value of the function is a real number.

EXAMPLES Find the domain of each function.

4 $f(x) = 2x - 5$

Since multiplication (by 2) and subtraction are defined for all real numbers, the domain of $f(x)$ is all real numbers.

5 $g(x) = \frac{1}{x-2}$

$g(x)$ is not defined when $x - 2 = 0$. So, $x \neq 2$.
 The domain is $\{x \mid x \neq 2\}$.

6 $h(x) = \sqrt{x+6}$

$h(x)$ is not defined when $x + 6 < 0$. So $x + 6 \geq 0$, which occurs when $x \geq -6$.
 The domain is $\{x \mid x \geq -6\}$.

Try This Find the domain of each function.

g. $f(x) = x^2 + 2$

h. $g(x) = \frac{1}{x+5}$

i. $h(x) = \sqrt{x-1}$

PART 3 Graphing Functions

When the domain of a function has an infinite number of values, we graph the function by graphing some of the ordered pairs and connecting points.

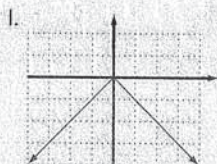
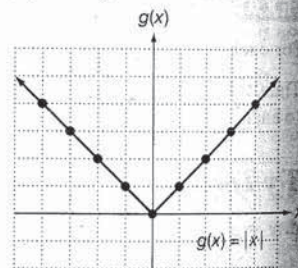
EXAMPLE 7 Graph the function $g(x) = |x|$. Then find the range.

We cannot list all of the ordered pairs for this function, since there is an infinite number of real numbers. Thus we begin by listing some values of the function.

x	0	1	-1	2	-2	3	-3
$g(x)$	0	1	1	2	2	3	3

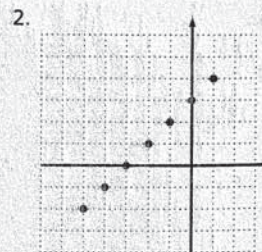
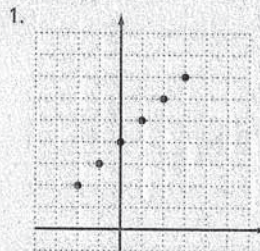
Now graph the ordered pairs. Connect the points to show all points whose coordinates are solutions of the equation $g(x) = |x|$.

Since the range consists of an infinite number of outputs, we use set-builder notation and the letter y . The range is $\{y \mid y \geq 0\}$.



$\{y \mid y \leq 0\}$

Exercises



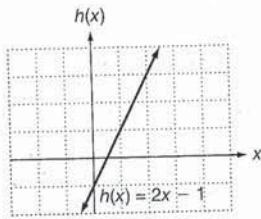
3. See Additional Answers.
 4. See Additional Answers.

see pp. 658-663

EXAMPLE 8 Graph the function $h(x) = 2x - 1$. Then find the range.

The domain of $h(x)$ is all real numbers. We recognize the form in which $h(x)$ is written as the equation of a line with slope 2 and y-intercept -1.

The range of $h(x)$ is all real numbers.



Try This Graph each function. Then find the range.

- j. $f(x) = |x| - 1$ k. $g(x) = -x + 4$ l. $h(x) = -|x|$

12-2 Exercises

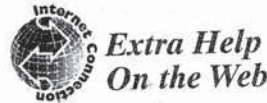
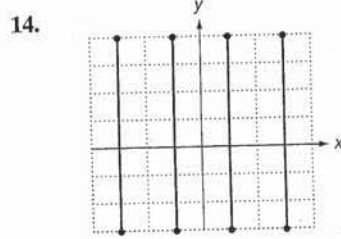
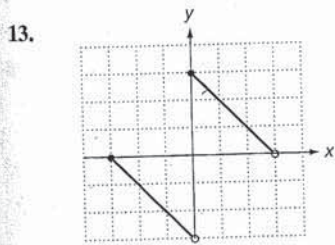
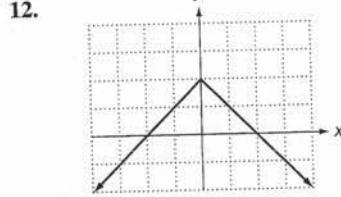
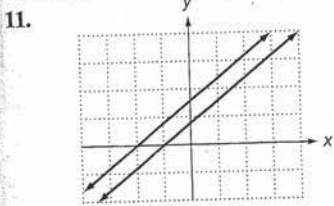
A
Graph each function.

- $f(x) = x + 4$ where the domain is $\{-2, -1, 0, 1, 2, 3\}$.
- $g(x) = x + 3$ where the domain is $\{-5, -4, -3, -2, -1, 0, 1\}$.
- $h(x) = 2x - 3$ where the domain is $\{-3, -1, 1, 3\}$.
- $f(x) = 3x - 1$ where the domain is $\{-1, 0, 2, 4, 6\}$.

Each relation below is a set of ordered pairs (x, y) that satisfy the given equation. Which of the relations are functions?

- $y = -x$
- $y = x^2$
- $x = |y| - 1$
- $x = y^2 + 2$
- $y = |x + 3|$
- $y = 6x + 9$

Which of the following are graphs of functions?



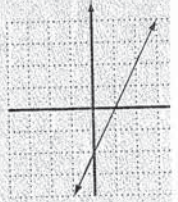
**Extra Help
On the Web**

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

3. PRACTICE/ASSESS

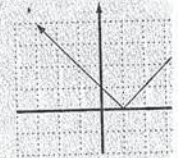
LESSON QUIZ

- Graph the function $f(x) = 2x - 1$. State the domain and the range.



The domain is all real numbers. The range is all real numbers.

- Graph the function $h(x) = -|x|$. State the domain and the range.



The domain is all real numbers. The range is $\{y \mid y \leq 0\}$.

Assignment Guide

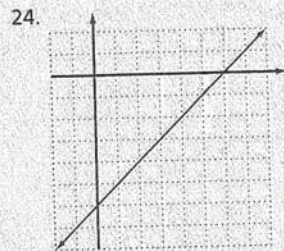
To provide flexible scheduling, this lesson can be split into parts.

- ▼ Core 1-14
- Extension 30
- ▼ Core 15-23
- Extension 33
- ▼ Core 24-29
- Extension 31, 32, 34-36

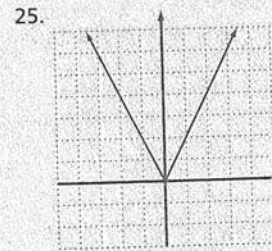
Use Mixed Review to monitor student progress.

- Yes
- Yes
- No
- No
- Yes
- Yes
- No
- Yes
- No
- Yes
- Yes
- No
- No
- $\{x \mid x \neq 0\}$
- All real numbers
- $\{x \mid x \geq 0\}$
- $\{x \mid x \neq 7\}$

- $\{x \mid x \geq -2\}$
- All real numbers
- $\{x \mid x \neq -3\}$
- All real numbers
- $\{x \mid x \leq 4\}$



Domain: all real numbers
Range: all real numbers



Domain: all real numbers
Range: $\{y \mid y \geq 0\}$



Practice Multiple Choice

Choose the best answer.

1. Which ordered pair could be removed from the relation below to change it into a function?

$\{(2, 3), (3, 2), (2, 5), (5, 4), (1, 6), (6, -1), (25, 31)\}$

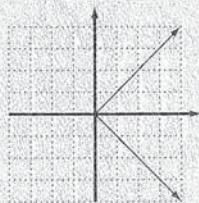
- A (2, 5)
- B (25, 31)
- C (6, -1)
- D (3, 2)

2. For the graph $y = |x| + 1$, which of the following statements is true?

- F The graph represents a function whose range is all real numbers.
- G The graph represents a function that assigns to each member of the domain exactly one member in the range.
- H The graph does not represent a function.
- J The graph represents neither a function nor a relation.

- 1. A; Algebra 16.0
- 2. G; Algebra 18.0

32. Answers may vary. Sample:



33. A

Find the domain of each function.

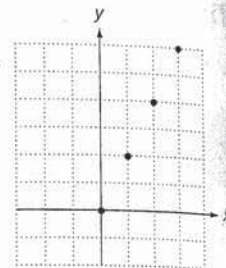
- 15. $f(x) = \frac{1}{3x}$
- 16. $g(x) = 10x$
- 17. $h(x) = \sqrt{x}$
- 18. $f(x) = \frac{5}{7-x}$
- 19. $g(x) = \sqrt{x+2}$
- 20. $h(x) = x^3$
- 21. $f(x) = \frac{1}{x+3}$
- 22. $g(x) = -2x - 9$
- 23. $h(x) = \sqrt{4-x}$

Graph each function. State the domain and the range.

- 24. $f(x) = x - 6$
- 25. $g(x) = 2|x|$
- 26. $h(x) = -2x + 1$
- 27. $f(x) = |x| - 3$
- 28. $g(x) = |x - 1|$
- 29. $h(x) = 4 - \frac{1}{2}x$

B

30. Here is the graph of a function. List the domain and the range of the function. Write an equation for this function.



31. Draw the graph of $y = \frac{1}{x}$. Find the domain and range of the relation. Is this the graph of a function?

32. **Critical Thinking** Sketch an absolute-value graph that is not a function.

33. **TEST PREP** Which of the functions below has 0 in its domain?

- A. $f(x) = \frac{1}{x+4}$
- B. $f(x) = \frac{1}{x} - 4$
- C. $f(x) = \sqrt{x-4}$
- D. $f(x) = \frac{1}{\sqrt{x}}$

34. **Mathematical Reasoning** When you graph a function by making a table and graphing ordered pairs, it is not always correct to connect the points you have graphed. Give an example of a function for which this is the case. (*Hint:* Consider a function for which the domain does not consist of all real numbers.)

Challenge

- 35. Describe the graph of any function of the form $g(x) = a|x| + b$.
- 36. For the function defined by $g(x) = a|x| + b$, what effect does changing the value of a have on the graph? What effect does changing the value of b have on the graph?

Mixed Review

Simplify. 37. $\frac{\sqrt{3}}{\sqrt{147}}$ 38. $\frac{\sqrt{20}}{\sqrt{45}}$ 39. $\frac{\sqrt{30}}{\sqrt{18}}$ 40. $\frac{\sqrt{7}}{\sqrt{xy}}$ 11-5

Divide. 41. $(x^2 - 9x + 20) \div (x - 4)$ 42. $(y^4 - 9) \div (y - 3)$ 10-9

Simplify. 43. $\frac{x^2 + 4x}{x + 3} - \frac{3x + 1}{x + 3} - \frac{5}{x + 3}$ 44. $\frac{3y}{y^2 - 25} - \frac{2}{y + 5}$ 10-4, 10-5

34. Answers may vary. Sample: $f(x) = \frac{1}{x}$ (See Exercise 31.)

35. \vee shaped, or \wedge if a is negative, with "point" on the y -axis b units up or down from the origin

36. If $|a|$ increases, the graph gets thinner; if b increases, the graph is moved upward.

Mixed Review

37. $\frac{1}{7}$

38. $\frac{2}{3}$

39. $\frac{\sqrt{15}}{3}$

40. $\frac{\sqrt{7xy}}{xy}$

41. $(x - 5)$

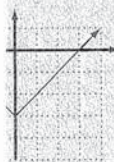
42. $y^3 + 3y^2 + 9y + 27 + \frac{72}{y-3}$

43. $(x - 2)$

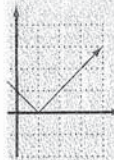
44. $\frac{y + 10}{y^2 - 25}$



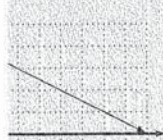
all real numbers
all real numbers



all real numbers
 $\{y \mid y \geq -3\}$

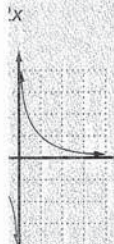


all real numbers
 $\{y \mid y \geq 0\}$



all real numbers
all real numbers

$\{0, 1, 2, 3\}$; Range $\{0, 2, 4, 6\}$;



$\{x \mid x \neq 0\}$
 $\{y \mid y \neq 0\}$; yes

PRactice/ASSESS

JIZ

Write the situation to an equation that represents a linear function. Then solve the equation.

1. A taxi ride costs \$1 starting fee plus \$0.75 per mile. How much does a 10-mile trip cost?

$$c = 1 + 0.75m$$

$$= \$8.50$$

2. A phone service costs \$8 per month, plus \$0.10 per message unit. How much is the monthly bill if 40 message units are used?

$$= 8.00 + 0.10m$$

$$= \$12.00$$

Performance Guide

Objectives 1–12
Connections 13–16, Connections

Use the Review to maintain skills.



Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.

www.phschool.com

We can now translate to an equation.

Amount needed is half of the capacity.

$$\begin{array}{ccccccc} \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow \\ y & = & \frac{1}{2} & \cdot & x & & \end{array}$$

This equation shows that for each value of x in the relation there is exactly one value for y . Therefore, the relation is a function. We can rewrite the equation to show this.

$$f(x) = \frac{1}{2}x$$

We can substitute 15 for x to find the amount of antifreeze needed.

$$\begin{aligned} f(15) &= \frac{1}{2} \cdot 15 \\ &= 7.5 \end{aligned}$$

Find the ANSWER and CHECK

The amount of antifreeze needed is 7.5 L.

This is half of 15 L. The answer is reasonable.

Try This Write a linear function. Then solve.

- a. Frank's Baby-Sitting Service charges \$3.50 plus \$3.65 per hour. What is the cost of a nine-hour baby-sitting job?

12-3 Exercises

A

Write a linear function describing each situation. Use the function to solve the problem.

- The Triad Truck Rental Company charges \$35 per day plus 21¢ per mile. Find the cost of renting a truck for a one-day trip of 340 miles.
- A cable television provider charges \$27 per month, plus \$3.95 for any movies ordered. Find the total charge for a month in which 3 movies are ordered.
- The cost of renting a chain saw is \$5.90 per hour plus \$6.50 for a can of gas. Find the cost of using the chain saw for 7.5 hours.
- The airport parking garage charges \$1.25 for the first hour and 70¢ for each additional hour. Find the cost of parking for 18 hours.
- The cost of renting a floor waxer is \$4.25 per hour plus \$5.50 for the wax. Find the cost of waxing a floor if the time involved was 4.5 hours.
- Sally rents a compact car for \$29 per day plus 19¢ per mile. Compute the cost of a one-day trip of 280 miles.

548 Chapter 12 Relations and Functions

Try This

- a. $c(h) = 3.50 + 3.65h$;
 $c(9) = \$36.35$

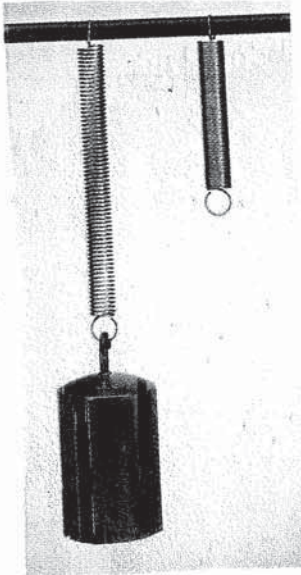
Exercises

- $c(k) = 35 + 0.21k$;
 $c(340) = \$106.40$
- $c(m) = 27.00 + 3.95m$;
 $c(3) = \$38.85$
- $c(h) = 6.50 + 5.90h$;
 $c(7.5) = \$50.75$
- $c(h) = 0.70(h - 1) + 1.25$;
 $c(18) = \$13.15$
- $c(h) = 5.50 + 4.25h$;
 $c(4.5) = \$24.63$
- $c(m) = 29 + 0.19m$;
 $c(280) = \$82.20$
- $L(w) = \frac{1}{3}w + 40$;
 $L(15) = 45$ cm
- $L(w) = \frac{1}{5}w + 60$;
 $L(20) = 64$ cm
Photo caption: 4 cm
- \$8.70; \$20.10
- \$10.50; \$30.05
- \$12.50
- \$135.00; 212 miles
- A horizontal line
- Answers may vary.
If you make a yearly salary, you are paid a constant amount each month regardless of the number of hours you work. Say you make \$600 a month, then $f(h) = 600$ no matter what h (hours worked) is. This of course differs from a job where you are paid by the hour, which is not a constant function.

7. A 40-cm spring will stretch (in cm) one third the weight (in kg) attached to it. How long will the spring be if a 15-kg weight is attached?
8. A 60-cm spring will stretch (in cm) one fifth the weight (in kg) attached to it. How long will the spring be if a 20-kg weight is attached?

B

9. A woman's phone bill is based on 15¢ per message unit charge plus a base charge. Her July bill was \$18 and included 62 message units. Find the base charge. Her August bill included 76 message units. How much was her August bill?
10. The city's water department charges 23¢ per kiloliter (kL) plus a fixed charge. Mr. Ahmed's water bill for the use of 70 kL was \$26.60. Find the fixed charge. The next month he used 85 kL. How much was his bill?
11. A parking garage in the city charges \$2.00 for the first hour and \$1.50 for each additional hour. Saturday and Sunday, the rates are decreased by 50%. How much does it cost to park a car from 5 p.m. on Friday until 2 a.m. on Saturday?
12. A truck rental company charges \$41 per day plus 24¢ for each mile over 100 miles. Find the cost of renting a truck for three days during which it is driven 150 miles. How far can you drive in three days and keep the cost under \$150?
13. A constant function is any function that can be described by an equation of the form $y = k$. Sketch the graph of a constant function.
14. **Critical Thinking** Make up a real-world example of a constant function.



This 5-cm spring stretches (in cm) 0.02 times the weight (in grams) attached to it. How far does this spring stretch with a 200 g weight attached to it?

#9 $.15 \times 62 =$

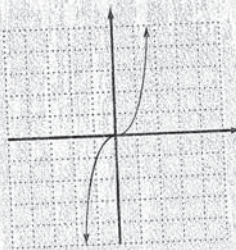
Challenge

15. Draw the graph of the function described by $f(t) = t^3$.
16. A restaurant has some fixed operating costs and some that vary with the number of people in attendance. If 4000 people visit the restaurant in one month, the operating costs are \$1300. If 2800 people visit the restaurant in one month, the operating costs are \$970. What would be the operating costs for this restaurant in a month where only 1500 people visit the restaurant?

Mixed Review

- Add or subtract. 17. $3\sqrt{8} + 2\sqrt{2}$ 18. $\sqrt{24} - \sqrt{6}$ 11-6
- Write using scientific notation. 19. 5,470,000 20. 0.0034709 5-4
- Simplify. 21. $\frac{6(x+1)}{5x} \cdot \frac{35x}{21x+21}$ 22. $\frac{(y+2)^3}{(y-2)^2} \div \frac{(y+2)^3}{(y-2)^2}$ 11-4, 11-5
- Solve and graph. 23. $|3x| > 12$ 24. $|2x| \geq 6$ 25. $|x-1| > 3$ 9-4

15.

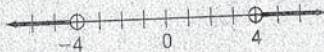


16. $c(p) = 200 + 0.275p$;
 $c(1500) = \$612.50$

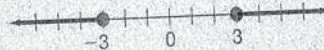
Mixed Review

17. $8\sqrt{2}$
 18. $\sqrt{6}$
 19. 5.47×10^6
 20. 3.4709×10^{-3}
 21. 2
 22. 1

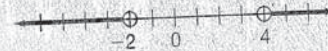
23. $x > 4$ or $x < -4$



24. $x \geq 3$ or $x \leq -3$

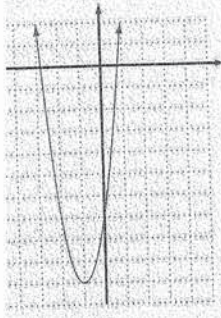


25. $x > 4$ or $x < -2$



TICE/ASSESS

11Z
 vertex and the axis of symmetry. Then graph the function $y = x^2 + 6x - 7$. The vertex is $(-1, -10)$. The axis of symmetry is the line $x = -1$.



Assignment Guide
 Section 1-28
 Extension 29-36
 Mixed Review to maintain skills.

page 553.
12S
 Graph of a parabola opening downwards on a coordinate plane. The vertex is at (0, 3). The x-axis and y-axis are shown with grid lines.

Domain: all real numbers
 Range: $\{y \mid y \leq 0\}$

Graph of a parabola opening upwards on a coordinate plane. The vertex is at (0, 0). The x-axis and y-axis are shown with grid lines.

Domain: all real numbers
 Range: $\{y \mid y \geq 0\}$

Graph of a parabola opening upwards on a coordinate plane. The vertex is at (0, 0). The x-axis and y-axis are shown with grid lines.

Domain: all real numbers
 Range: $\{y \mid y \geq 0\}$

Substitute the x -coordinate of the vertex into the original equation. Solve for y .

$$y = x^2 + 2x + 3$$

$$= -(1)^2 + 2(1) + 3$$

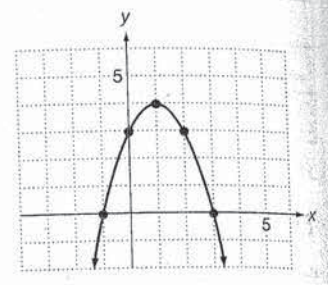
$$y = 4$$

vertex: $(1, 4)$

The axis of symmetry is the line $x = 1$.
 Now choose points on both sides of the vertex and graph the parabola.

x	y
-1	0
0	3
1	4
2	3
3	0

$y \leq 4$



Try This Graph each function.

b. $f(x) = -2x^2 + 4x + 1$ c. $g(x) = x^2 - 3x + 1$

In Example 2, the x -intercepts of the graph of $f(x) = -x^2 + 2x + 3$ are -1 and 3 . We can also find the x -intercepts by setting the function equal to zero and solving:

$$-x^2 + 2x + 3 = 0$$

$$(x + 1)(-x + 3) = 0 \quad \text{Factoring}$$

$$x = -1, 3$$

Try This Find the x -intercepts of each function.

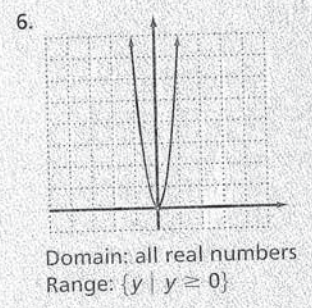
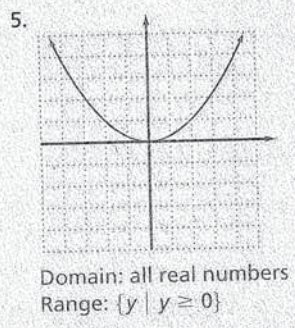
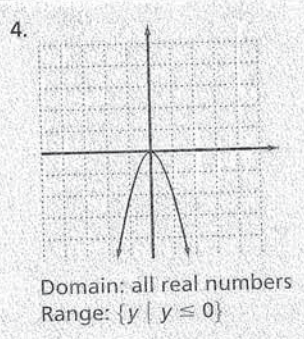
d. $f(x) = x^2 - 5x + 6$ e. $g(x) = -x^2 + 3x + 4$

Internet Connection
Extra Help On the Web
 Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

12-4 Exercises

A
 Graph each function. State the domain and range.

- $f(x) = -3x^2$
- $f(x) = 3x^2$
- $f(x) = 2x^2$
- $f(x) = -2x^2$
- $f(x) = \frac{1}{4}x^2$
- $f(x) = 8x^2$
- $f(x) = x^2 - 3$
- $f(x) = -x^2 + 3$
- $f(x) = 2x^2 - 1$



Find the vertex by $-\frac{b}{2a}$

Graph each function.

- 10. $f(x) = x^2 + x - 6$
- 11. $f(x) = -x^2 + x - 1$
- 12. $f(x) = 8 - x - x^2$
- 13. $y = x^2 + 10x + 25$
- 14. $y = 2x^2 + 4x - 1$
- 15. $y = x^2 + 5$

Find the x-intercepts of the graph of each function.

- 16. $f(x) = x^2 - 3x - 10$
- 17. $y = x^2 + 2x$
- 18. $g(x) = -x^2 + 10x - 21$
- 19. $y = x^2 - 8x + 16$
- 20. $y = 2x^2 - 9x + 10$
- 21. $h(x) = -x^2 - 11x - 18$

B

- 22. Graph $f(x) = x^2 - 6x + 8$. What are the roots of $x^2 - 6x + 8$?
- 23. Graph $f(x) = -x^2 + x + 2$. What are the roots of $-x^2 + x + 2$?
- 24. For the graph of a linear function $f(x) = mx + b$, we know the y-intercept is b . Examine the graphs of the functions in Exercises 7-15. Describe the y-intercept of the graph of a quadratic function.
- 25. The graph of a quadratic function opens upward or opens downward. Examine the graphs in Exercises 7-15. What is true about functions whose graphs open upward? Downward?
Determine if the graph of each equation opens upward or downward.
 - a. $y = 2x^2$
 - b. $y = -x^2 + 8$
 - c. $y = 5 + 3x - x^2$
- 26. The graph of each quadratic function has either a high point (maximum) or low point (minimum). Examine the graphs in Exercises 7-15. What types of graphs have a maximum or minimum?
- 27. Determine the minimum or maximum point for each equation. Indicate whether it is a minimum or maximum.
 - a. $y = 3x^2 - 4$
 - b. $y = -x^2 + 7$
 - c. $y = 2 + 3x - 6x^2$
- 28. Find the axis of symmetry for each parabola graphed in Exercises 7-15.

When a projectile is thrown into the air with an initial vertical velocity of r feet per second, its distance (d), in feet, above its starting point t seconds after it is thrown is approximately

$$d = rt - 16t^2$$

For a projectile launched from the ground with an initial upward velocity of 96 feet per second, the function is

$$d = 96t - 16t^2$$

Use the graph of this quadratic function to answer Exercises 29-31.

- 29. How many seconds after launch is the projectile 128 feet above the ground?
- 30. When does the projectile reach its maximum height? What is this height?
- 31. How many seconds after launch does the projectile return to the ground?

Vertex # 10

$$-\frac{1}{2(1)} = -\frac{1}{2}$$

$$\left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 6 = \frac{1}{4} - \frac{1}{2} - 6 = -6\frac{1}{4}$$

Vertex: $\left(-\frac{1}{2}, -6\frac{1}{4}\right)$

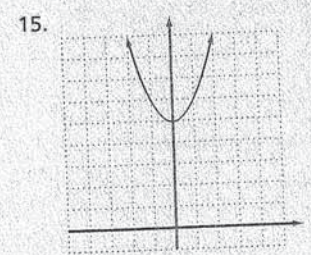
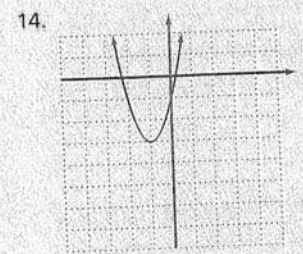
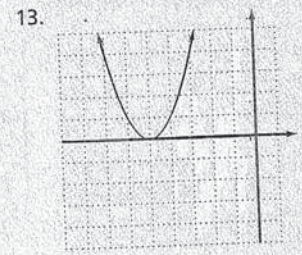
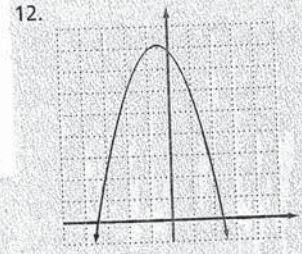
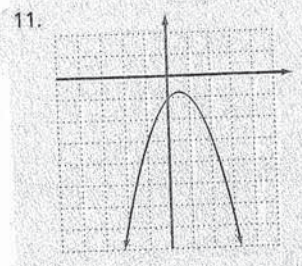
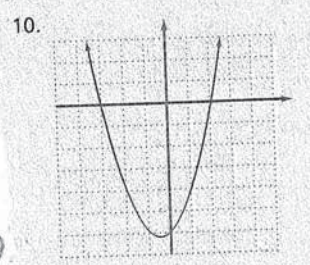
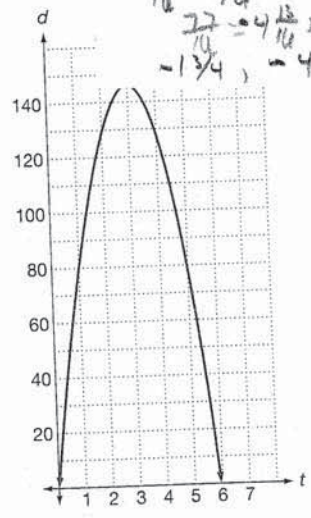
Vertex # 11

$$\frac{1}{2(-1)} = -\frac{1}{2}$$

$$-\frac{1}{4} - \frac{1}{2} - 1 = -1\frac{3}{4}$$

$$-\frac{49}{16} = -\frac{49}{16}$$

$$-\frac{27}{10} = -\frac{27}{10}$$

$$-\frac{1}{4}, -\frac{13}{10}$$


7.

Domain: all real numbers
Range: $\{y \mid y \geq -3\}$

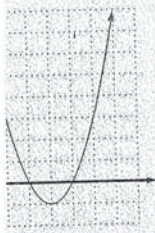
8.

Domain: all real numbers
Range: $\{y \mid y \leq 3\}$

9.

Domain: all real numbers
Range: $\{y \mid y \geq -1\}$

16-31. See page 556.



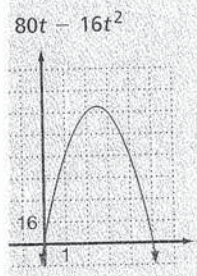
Intercepts of the graph are 2
The roots of the polynomial are



Intercepts of the graph are
2. The roots of the polynomial
and 2.

The function is in standard form,
intercept is the constant term.

$t < 0$
ird
ward
ward
that open upward have a
m point. Graphs that open
ard have a maximum point.
4) min. pt.
max. pt.
9) max. pt.
 $x = 0, x = 0, x = -\frac{1}{2}, x = \frac{1}{2},$
 $x = -5, x = -1, x = 0$



ft, 96 ft
s
ft
increases, the graph narrows.
3.4



Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

32. **Multi-Step Problem** To go over a tree, a golfer uses a 7-iron to hit the ball with an initial upward velocity of 80 feet per second.
- Write a quadratic function that gives the height of the ball t seconds after it is hit.
 - Graph the function you found in part (a).
 - How high will the ball be after 1 second? 2 seconds?
 - After how many seconds will the ball be at its maximum height?
 - What is the approximate maximum height the ball will reach?
33. **Critical Thinking** Use the same set of axes. Graph $y = x^2$, $y = 2x^2$, $y = 3x^2$, and $y = \frac{1}{2}x^2$. Describe the change in the graph of a quadratic function $y = ax^2$ as $|a|$ changes.

Challenge

34. Graph the equation $y = x^2 - x - 6$. Use your graph to approximate the solutions of $x^2 - x - 6 = 2$. (Hint: Graph $y = 2$ on the same set of axes as your graph of $y = x^2 - x - 6$.)
35. Graph $y = 2x^2 - 4x + 7$. Use the graph to approximate the solutions of $2x^2 - 4x + 7 = 0$. (Hint: Find the values of x when y is 0.)
36. What is the largest rectangular area that can be enclosed with 36 ft of fence?

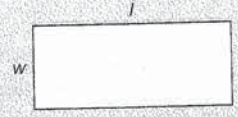
Mixed Review

- Write using decimal notation. 37. 5.115×10^{-7} 38. 4.7933×10^5
- Solve. 39. $\sqrt{x} = 17$ 40. $\sqrt{2y - 1} = 9$ 41. $11 - \sqrt{x + 1} = 2$
42. $2\sqrt{2x} = 20$ 43. $\sqrt{3x + 4} = 5$ 44. $5 + \sqrt{2x - 1} = 8$
- Add or subtract. 45. $\sqrt{\frac{5}{7}} - \sqrt{\frac{7}{5}}$ 46. $x - \frac{x}{\sqrt{x}}$ 47. $\sqrt{\frac{2}{5}} + \sqrt{\frac{1}{15}}$
- Find the indicated outputs for these functions.
48. $f(m) = 2m - 1$; find $f(-1)$, $f(4)$, $f(2)$, and $f(-11)$
49. $g(x) = -3x + 5$; find $g(2)$, $g(-2)$, $g(0)$, and $g(5)$
- Simplify. 50. $\sqrt{36}$ 51. $\sqrt{(x - 5)^2}$ 52. $\sqrt{(-5y)^2}$ 53. $\sqrt{243m^3}$

Connections: Geometry

What is the largest rectangular area that can be enclosed with 16 ft of fence?

$$\begin{aligned} 2w + 2l &= 16 \\ w + l &= 8 \\ w &= 8 - l \end{aligned}$$



Substitute for w in the formula for the area of a rectangle: $A = lw$.
Graph the resulting quadratic function and find its maximum.

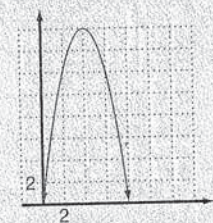
35. If the graph of the function does not cross the x -axis, then the equation has no real-number solution.
36. $81t^2$

Mixed Review

37. 0.0000005115
38. 479,330
39. 289
40. 41
41. 80
42. 50
43. 7

44. 5
45. $-\frac{2\sqrt{35}}{35}$
46. $x - \sqrt{x}$
47. $\frac{3\sqrt{10} + \sqrt{15}}{15}$
48. -3, 7, 3, -23
49. -1, 11, 5, -10
50. 6
51. $|x - 5|$
52. $5|y|$
53. $9|m|\sqrt{3m}$

Connections: Geometry



Maximum at (4, 16), so the largest area that can be enclosed is 16 ft^2 .

12-5 Exercises



Extra Help
On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A
Find an equation of variation where y varies directly as x , and the following are true.

- $y = 28$ when $x = 7$
- $y = 30$ when $x = 8$
- $y = 0.7$ when $x = 0.4$
- $y = 0.8$ when $x = 0.5$
- $y = 400$ when $x = 125$
- $y = 630$ when $x = 175$
- $y = 200$ when $x = 300$
- $y = 500$ when $x = 60$

Solve.

- A person's paycheck (p) varies directly as the number (h) of hours worked. For working 15 hours, the pay is \$236.25. Find the pay for 35 hours of work.
- The number (b) of bolts a machine can make varies directly as the time it operates. It can make 6578 bolts in 2 hours. How many can it make in 5 hours?
- The number (n) of servings of meat that can be obtained from a turkey varies directly as its weight (w). A turkey weighing 22 lb yields 40 servings of meat. How many servings can be obtained from a 14-lb turkey?
- The number (n) of servings of meat that can be obtained from round steak varies directly as the weight (w). From 9 kg of round steak one can get 70 servings of meat. How many servings can one get from 12 kg of round steak?
- The weight (M) of an object on the moon varies directly as its weight (E) on Earth. A person who weighs 115 lb on Earth weighs 19 lb on the moon. How much would a person who weighs 150 lb on Earth weigh on the moon?
- The weight (M) of an object on Mars varies directly as its weight (E) on Earth. A person who weighs 115 lb on Earth weighs 44 lb on Mars. How much would a person who weighs 150 lb on Earth weigh on Mars?
- The number of kg of water (W) in a human body varies directly as the total body weight (B). A person who weighs 75 kg contains 54 kg of water. How many kilograms of water are in a person weighing 95 kg?
- The amount (A) that a family gives to charity varies directly as its income (I). Last year, the family earned \$32,000 and gave \$2560 to charity. How much will they give if they make \$42,000 this year?
- The distance between two cities is 1500 miles. They are shown to be 6 in. apart on a globe. What is the actual distance between two cities that are shown to be 10 inches apart on the globe?
- The volume inside a balloon varies directly as the temperature. The volume of the balloon is 3 L at a temperature of 300° Kelvin. What is the volume if the temperature rises to 350° Kelvin?

3. PRACTICE/ASSESS

LESSON QUIZ

- Find the equation of variation varies directly as x , and $y = 7$ when $x = 2$.
 $y = \frac{7}{2}x$
- A salesman's income varies directly as the dollar amount of sales per year. The salesman made \$3000 of income on sales of \$12,000. How much income can he make on sales of \$17,000?

$$I = \frac{1}{4}S$$

\$4250

Assignment Guide

To provide flexible scheduling, this lesson can be split into parts.

▼ Core 1–8
Extension 24–28

▼ Core 9–18
Extension 19–23, 29–31

Use Mixed Review to maintain

Exercises

- $y = 4x$
- $y = 3.75x$
- $y = 1.75x$
- $y = 1.6x$
- $y = 3.2x$
- $y = 3.6x$
- $y = \frac{2}{3}x$
- $y = \frac{25}{3}x$
- \$551.25
- 16,445 bolts
- Approx. 25 servings
- Approx. 93 servings
- Approx. 25 lb
- Approx. 57 lb
- 68.4 kg
- \$3360
- 2500 miles
- 3.5 L

B

Which of the following vary directly?

19. the amount of a gas in a tank in liters and the amount in gallons
20. the temperature in Fahrenheit degrees and in Celsius
21. the price per pound of carrots and the number of pounds
22. the total price of tomatoes and the number of pounds
23. a number and its reciprocal

Write an equation of direct variation for each situation.

24. The perimeter (P) of an equilateral polygon varies directly as the length of a side (S).
25. The circumference of a circle (C) varies directly as the radius (r).
26. The number of bags (B) of peanuts sold at a baseball game varies directly as the number (N) of people in attendance.
27. The cost (C) of building a new house varies directly as the area (A) of the floor space of the house.
28. **Critical Thinking** Describe the graph of an equation of direct variation ($y = kx$). What is the relationship between k , the constant of variation and the graph of the equation?

Challenge

Write an equation of variation to describe these situations.

29. In a stream, the amount of salt (S) carried varies directly as the sixth power of the speed (V) of the stream.
30. The square of the pitch (P) of a vibrating string varies directly as the tension (t) on the string.
31. The surface area (A) of a sphere varies directly as the square of the radius (r).

Mixed Review

Determine the replacements for the variables that give real numbers.

32. $\sqrt{5t}$
33. $\sqrt{3x^2}$
34. $\sqrt{a-7}$
35. $\sqrt{3m-1}$
36. $\sqrt{y^2+3}$
37. $\sqrt{x^2-4x+4}$
38. $\sqrt{a^2+6a+9}$ 11-1

Rationalize the denominator.

39. $\frac{\sqrt{y}}{\sqrt{xy}}$
40. $\frac{\sqrt{15x^3y^2}}{\sqrt{5x^2y}}$
41. $\frac{\sqrt{18m^2n}}{\sqrt{16n}}$ 11-5

In a right triangle, find the length of the side not given.

42. $a = 7, c = 14$
43. $a = 10, b = 10$
44. $b = 12, c = 15$ 11-7

Find the indicated outputs for these functions.

45. $f(t) = 2t^2 - 10$; find $f(1), f(-2), f(5), f(-4)$
46. $g(c) = |c| - 3$; find $g(1), g(5), g(-3), g(-7)$ 12-1

Exercises

19. Yes
20. No
21. No
22. Yes
23. No
24. $P = kS$ (k = number of sides)
25. $C = kr$ ($k = 2\pi$)
26. $B = kN$
27. $C = kA$
28. The graph of an equation of direct variation is a line through the origin with a slope equal to k .

29. $S = kV^6$
30. $P^2 = kt$
31. $A = kr^2$

Mixed Review

32. $t \geq 0$
33. Any real number
34. $a \geq 7$
35. $m \geq \frac{1}{3}$
36. Any real number
37. Any real number
38. Any real number

39. $\frac{\sqrt{x}}{x}$

40. $\sqrt{3xy}$

41. $\frac{3|m|\sqrt{2}}{4}$

42. $7\sqrt{3}$

43. $10\sqrt{2}$

44. 9

45. -8, -2, 40, 22

46. -2, 2, 0, 4

12-6 Exercises



Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A
Find an equation of variation where y varies inversely as x . One pair of values is given.

- | | |
|--|--|
| 1. $y = 25$ when $x = 3$ | 2. $y = 45$ when $x = 2$ |
| 3. $y = 8$ when $x = 10$ | 4. $y = 7$ when $x = 10$ |
| 5. $y = 0.125$ when $x = 8$ | 6. $y = 6.25$ when $x = 0.16$ |
| 7. $y = 42$ when $x = 25$ | 8. $y = 42$ when $x = 50$ |
| 9. $y = 0.2$ when $x = 0.3$ | 10. $y = 0.4$ when $x = 0.6$ |
| 11. $y = 0.8$ when $x = 4$ | 12. $y = 80$ when $x = 0.7$ |
| 13. $y = \frac{2}{5}$ when $x = \frac{5}{2}$ | 14. $y = \frac{4}{3}$ when $x = \frac{3}{2}$ |

Solve.

- It takes 16 hours for 2 people to resurface a gym floor. How long would it take 6 people to do the job?
- It takes 4 hours for 9 cooks to prepare a school lunch. How long would it take 8 cooks to prepare the lunch?
- The volume (V) of a gas varies inversely as the pressure (P) upon it. The volume of a gas is 200 cubic centimeters (cm^3) under a pressure of 32 kg/cm^2 . What will be its volume under a pressure of 20 kg/cm^2 ?
- The current (I) in an electrical conductor varies inversely as the resistance (r) of the conductor. The current is 2 amperes when the resistance is 960 ohms. What is the current when the resistance is 540 ohms?
- The time (t) required to empty a tank varies inversely as the rate (r) of pumping. A pump can empty a tank in 90 minutes at the rate of 1200 L/min. How long will it take the pump to empty the tank at 2000 L/min?
- The height (H) of triangles of fixed area varies inversely as the base (B). Suppose the height is 50 cm when the base is 40 cm. Find the height when the base is 8 cm. What is the fixed area?

- B**
Write an equation of inverse variation for each situation.
- The cost per person (C) of chartering a fishing boat varies inversely as the number (N) of persons sharing the cost.
 - The number (N) of revolutions of a tire rolling over a given distance varies inversely as the circumference (C) of the tire.
 - The amount of current (I) flowing in an electric circuit varies inversely with the resistance (R) of the circuit.
 - The intensity of illumination (I) from a light source varies inversely as the square of the distance (d) from the source.

3. PRACTICE/ASSESS

LESSON QUIZ

- Find the equation of variation y varies inversely as x , and $y = 3$ when $x = 2$.
 $y = \frac{6}{x}$
- It takes 3 people 5 hours to sort 100 pounds of coffee beans. How long will it take 8 people to do the job?
 $t = \frac{15}{p}$
It will take 1.875 hours.

Assignment Guide

To provide flexible scheduling, this lesson can be split into parts.

- ▼ Core 1–14
Extension 27, 28
- ▼ Core 15–25
Extension 26, Connections

Use Mixed Review to maintain skills.

Exercises

- | | | |
|-------------------------|--------------------------|--------------------------------|
| 1. $y = \frac{75}{x}$ | 9. $y = \frac{0.06}{x}$ | 18. $3\frac{5}{9}$ amp |
| 2. $y = \frac{90}{x}$ | 10. $y = \frac{0.24}{x}$ | 19. 54 min |
| 3. $y = \frac{80}{x}$ | 11. $y = \frac{3.2}{x}$ | 20. 250 cm; 1000 cm^2 |
| 4. $y = \frac{70}{x}$ | 12. $y = \frac{56}{x}$ | 21. $C = \frac{k}{N}$ |
| 5. $y = \frac{1}{x}$ | 13. $y = \frac{1}{x}$ | 22. $N = \frac{k}{C}$ |
| 6. $y = \frac{1}{x}$ | 14. $y = \frac{2}{x}$ | 23. $I = \frac{k}{R}$ |
| 7. $y = \frac{1050}{x}$ | 15. $5\frac{1}{3}$ hr | 24. $I = \frac{k}{d^2}$ |
| 8. $y = \frac{2100}{x}$ | 16. $4\frac{1}{2}$ hr | |
| | 17. 320 cm^3 | |

Exercises

25. B
 26. If h is halved;
 it r is halved.

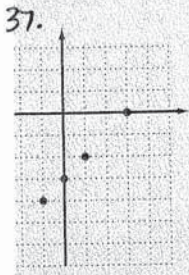
27. $F = \frac{ks^2m}{r}$

28. $L = \frac{kwt^2}{d}$

Mixed Review

- 1. $|x-5| \sqrt{2}$
- 2. $|a|c^2 \sqrt{a}$
- 1. 18
- 2. 33
- 3. NO
- 4. YES
- 25. YES
- 36. NO

view



25. **TEST PREP** Which one of the following varies inversely?
 A. the cost of sending a letter within the U.S. and the distance it travels
 B. a runner's speed in a race and the time it takes to run the race
 C. the area of a circle and the radius
 D. the weight of a turkey and the cooking time
26. **Critical Thinking** Suppose y varies inversely as x . If the value of x is doubled, what happens to the value of y ? If the value of y is doubled, what happens to the value of x ?

Challenge

- Write an equation of variation for each of the following.
27. The force (F) needed to keep a car from skidding on a curve varies directly as the square of the car's speed (S) and its mass (m) and inversely as the radius of the curve (r).
28. For a horizontal beam supported at both ends, the maximum safe load (L) varies directly as its width (w) and the square of its thickness (t) and inversely as the distance (d) between the supports.

Mixed Review

- Simplify. 29. $\sqrt{2x^2 - 20x + 50}$ 30. $\sqrt{ac^3} \sqrt{a^2c}$ 11-3, 11-4
- Solve. 31. $\sqrt{x-2} = 4$ 32. $7 - \sqrt{x+3} = 1$ 11-9
- Determine whether the given point is a solution of $3x + 4y > 2$.
33. $(-3, 1)$ 34. $(2, 0)$ 35. $(0, 1)$ 36. $(-1, 1)$ 9-5
- Graph the function. 37. $f(x) = x - 3$ where the domain is $\{-3, -1, 0, 1, 3\}$ 38. $g(x) = 2x - 5$ 39. $h(x) = |x + 1|$ 12-2

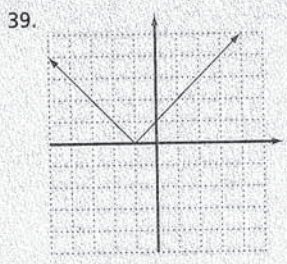
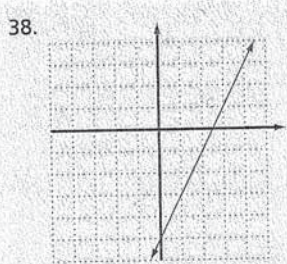
Connections: Reasoning

In each set of figures, the first two have a particular relationship. Which figure ($a, b, c,$ or d) has the same relationship to the third figure?

1. a. b. c. d.

2. a. b. c. d.

3. a. b. c. d.



Connections: Reasoning

- 1. d
- 2. d
- 3. a



Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

12-7 Exercises

A

Find an equation of joint variation. Then solve for the missing value.

- r varies directly as s and t . One set of values is $r = 28$, $s = 7$, $t = 8$. Find r when $s = 12$ and $t = 9$.
- m varies jointly as n and p . One set of values is $m = 86.4$, $n = 9$, and $p = 12$. Find m when $n = 20$ and $p = 6.5$.
- q varies jointly as r and s . One set of values is $q = 2.4$, $r = 0.6$, and $s = 0.8$. Find q when $r = 1.6$ and $s = 0.1$.
- a varies jointly as b and c . One set of values is $a = 1$, $b = 5$, and $c = 0.2$. Find a when $b = 2.4$ and $c = 0.01$.
- x varies jointly as w , y , and z . One set of values is $x = 18$, $w = 2$, $y = 6$, and $z = 5$. Find x when $w = 5$, $y = 12$, and $z = 3$.

Find an equation of combined variation. Then solve for the missing value.

- a varies directly as b and inversely as c . One set of values is $a = 14$, $b = 7$, and $c = 3$. Find a when $b = 4$ and $c = 8$.
- m varies directly as n and inversely as p . One set of values is $m = 5$, $n = 2$, and $p = 4$. Find m when $n = 10$ and $p = 50$.
- w varies directly as x and inversely as y . One set of values is $w = 3$, $x = 9$, and $y = 2$. Find w when $x = 15$ and $y = 5$.
- p varies directly as q and inversely as r . One set of values is $p = 0.8$, $q = 8$, and $r = 7$. Find p when $q = 12$ and $r = 3$.
- u varies directly as v and inversely as w . One set of values is $u = 5.75$, $v = 2.3$, and $w = 0.6$. Find u when $v = 0.5$ and $w = 0.8$.

B

- A varies jointly as b and c and inversely as d . One set of values is $A = 0.525$, $b = 6$, $c = 7$, and $d = 8$. Find A when $b = 12$, $c = 7$, and $d = 3$.
- W varies jointly as x and y and inversely as z . One set of values is $W = 112$, $x = 4$, $y = 8$, and $z = 0.2$. Find W when $x = 8$, $y = 10$ and $z = 4$.
- The volume of a cone varies jointly as the height of the cone and the area of the base. The volume of a cone with height 15 cm and base 28 cm^2 is 140 cm^3 . Find the volume of a cone with height 7 cm and base 12 cm^2 .
- A pitcher's earned run average (a) varies directly as the number of earned runs (r) allowed and inversely as the number of innings (i) pitched. Joe Price had an earned run average of 2.55. He gave up 85 earned runs in 300 innings. About how many earned runs would he have given up had he pitched 353 innings with the same average?

10. $u = \frac{1.5v}{w}$; 0.9375

11. 2.8

12. 14

13. 28 cm^3

14. 100

15. 7.9 in.

16. 3 cm

17. $\frac{\pi}{4}$

18. a. Answers may vary. Sample:
A pyramid with sides 1 and height 3 has volume
 $= \frac{1}{3}(1)^2(3) = 1$. A pyramid with sides 2 and height 6 has volume
 $= \frac{1}{3}(2)^2(6) = 8$.

The volume of the larger pyramid is not 4 times the volume of the smaller pyramid.

- b. Answers may vary. Sample:
If a pyramid has a square base and you double the length of the sides of the base and double the height, the volume of the new pyramid will be eight times the volume of the original pyramid.

19. y is doubled.

20. y is quadrupled.

21. Approx. 51 mi/h

22. 10 m

Mixed Review

23. $\frac{4\sqrt{3}}{3}$

24. $\frac{\sqrt{5x}}{x}$

25. $\frac{\sqrt{y}}{y}$

26. $\frac{3\sqrt{y}}{y}$

TEST/ASSESS

IZ

equation of joint variation varies jointly as a and b , and when $a = 2$ and $b = 10$.

equation of combined variation where U varies directly as V and inversely as W , and $U = 3$ when $V = 4$ and $W = 5$.

Student Guide

- 1-5
- Section 11, 19
- 6-10
- Section 12, 20
- 13-18
- Section 21, 22

Read Review to maintain skills.

EXERCISES

1. $r = \frac{1}{2}st$; 54

2. $m = 0.8np$; 104

3. $q = 5rs$; 0.8

4. $a = bc$; 0.024

5. $x = \frac{3}{10}wyz$; 54

6. $a = \frac{kb}{c}$; 3

7. $m = \frac{10n}{p}$; 2

8. $w = \frac{2x}{3y}$; 2

9. $p = \frac{0.7q}{r}$; 2.8

Use the following relationship: The lateral surface area of a cylinder varies jointly as the radius and height of the cylinder. A cylinder with radius 0.8 in. and height 6 in. has a lateral surface area of 30.2 in^2 .

15. Find the height of a cylinder with radius 1.5 in. and lateral surface area 74.4 in^2 .
16. Find the radius of a cylinder with height 6 cm and lateral surface area 111.6 cm^2 .
17. **Critical Thinking** The area of a circle varies directly as the square of the length of its diameter. What is the constant of variation?
18. **Error Analysis** A student made the following incorrect statement. "If a pyramid has a square base and you double the length of the sides of the base and double the height, the volume of the new pyramid will be four times the volume of the original pyramid."
 a. Provide a counterexample to show that the statement is incorrect.
 b. Rewrite the statement with a corrected conclusion.

Challenge

19. Suppose y varies directly as x . What happens to the value of y if the value of x is doubled?
20. Suppose y varies directly as the square of x . What happens to the value of y if the value of x is doubled?
21. The stopping distance (d) of a car after the brakes are applied varies directly as the square of the speed (r). A car traveling 35 mi/h will travel about 106 ft after braking. What is the rate of a car that needs about 228 ft to stop?
22. The force of attraction of a body varies directly as its mass and inversely as the square of the distance from the body. A body of mass 10 kg has a force of attraction of 7.8 newtons at a distance of 20 m. How far away is the force of attraction of a 10 kg mass equal to 31.2 newtons?



Practice Multiple Choice

Choose the best answer.

1. A student mistakenly claimed that the graph of a linear equation with positive slope never enters the fourth quadrant. Which equation is a counterexample to this statement?
 A $y = 2x - 3$
 B $y = 2x + 3$
 C $y = 2$
 D $y = x + 1$
2. The distance on a map varies directly as the actual distance. If 1.5 inches on the map is 20 actual miles, how close are towns on the map which are 230 miles apart?
 F $7\frac{2}{3}$ in.
 G $3066\frac{2}{3}$ in.
 H 11.5 in.
 J 17.25 in.

1. A; Algebra 25.1
 2. J; Algebra 4.0

12-7 Joint and Combined Variation 569

Mixed Review

- Rationalize the denominator. 23. $\frac{\sqrt{16}}{\sqrt{3}}$ 24. $\frac{\sqrt{5}}{\sqrt{x}}$ 25. $\frac{\sqrt{x}}{\sqrt{xy}}$
26. $\frac{\sqrt{18}}{\sqrt{2y}}$ 27. $\frac{\sqrt{3x}}{\sqrt{12y}}$ 28. $\frac{\sqrt{x^4y}}{\sqrt{xy^2}}$ 29. $\frac{\sqrt{6c}}{\sqrt{8c^3}}$ 30. $\frac{\sqrt{12cd}}{\sqrt{3c^4d^4}}$ 11-5

In a right triangle, find the length of the hypotenuse.

31. $a = 10, b = 24$ 32. $a = 6, b = 12$ 33. $a = 24, b = 32$ 11-7

Solve. 34. $\sqrt{2x+3} = \sqrt{5x-6}$ 35. $-\sqrt{x} = 7$

36. $\sqrt{x+7} = 2\sqrt{x-5}$ 37. $\sqrt{x+30} = 3\sqrt{x-2}$ 11-9

Graph. 38. $h(x) = 3x - 2$ 39. $f(x) = x^2 + 2$ 12-2, 12-4

27. $\frac{\sqrt{xy}}{2y}$

28. $\frac{x\sqrt{xy}}{y}$

29. $\frac{\sqrt{3}}{2c}$

30. $\frac{2\sqrt{cd}}{c^2d^2}$

31. 26

32. $6\sqrt{5}$

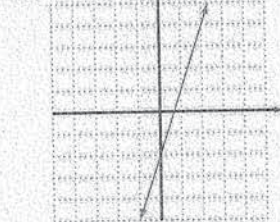
33. 40

34. 3

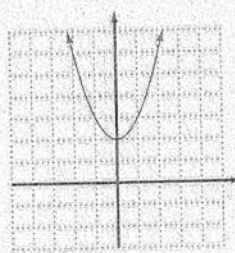
35. No real-number solution

36. 9

37. 6



39.



12 Chapter Wrap Up

12-1

A **relation** is a set of ordered pairs; a **function** is a relation that assigns to each member of the **domain** (the set of first coordinates) exactly one member of the **range** (the set of second coordinates).

Are the following relations functions?

- | | |
|---|---|
| <p>1. Domain Range</p> <p>-1 → 3</p> <p>0 → 4</p> <p>1 → 5</p> | <p>2. Domain Range</p> <p>-2 → 0</p> <p>5 → 1</p> <p>7 → 4</p> |
|---|---|

Find the indicated outputs for these functions.

- $f(x) = 3x - 4$; find $f(2)$, $f(0)$, and $f(-1)$
- $g(t) = |t| - 3$; find $g(3)$, $g(-5)$, and $g(0)$
- $h(x) = x^3 + 1$; find $h(-2)$, $h(0)$, and $h(-1)$

12-2

To graph a function, find some ordered pairs for the function, and plot the points. If the domain is the set of real numbers, then connect the points to show all the ordered pairs that belong to the function.

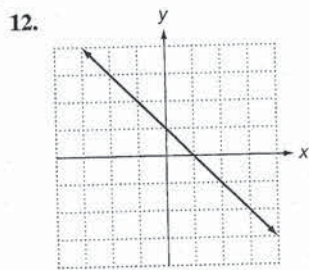
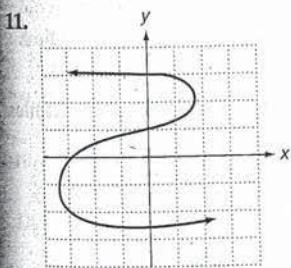
Graph each function with the given domain.

- $f(x) = 2x + 3$ where the domain is $(-2, -1, 0, 1, 2)$
- $g(x) = -3x$ where the domain is $(-4, -2, 0, 2, 4)$

Graph each function. State the domain and range.

- $g(x) = x + 7$
- $f(x) = x^2 - 3$
- $h(x) = 3|x|$

Which of the following are graphs of functions?

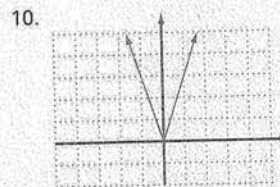
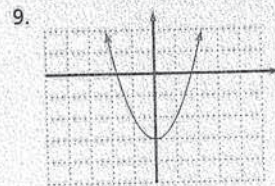
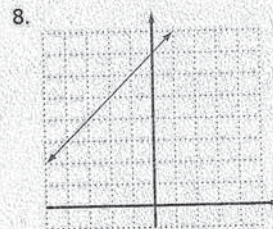
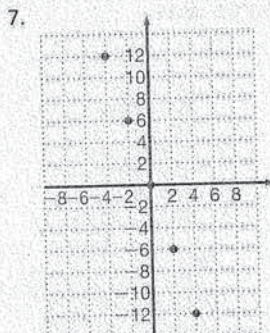
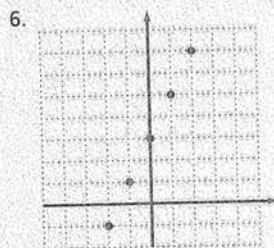


Key Terms

- axis of symmetry (p. 553)
- combined variation (p. 566)
- constant of variation (p. 557)
- direct variation (p. 557)
- domain (p. 536)
- function (p. 537)
- function values (p. 538)
- inputs (p. 537)
- inverse variation (p. 561)
- joint variation (p. 565)
- linear function (p. 547)
- outputs (p. 537)
- parabola (p. 552)
- quadratic function (p. 552)
- range (p. 536)
- relation (p. 536)
- vertex (p. 553)

Chapter 12 Wrap Up

- No
- Yes
- 2; -4; -7
- 0; 2; -3
- 7; 1; 0



- No
- Yes



Internet Activity On the Web

Look for extension problems for this chapter at the Prentice Hall Web site. www.phschool.com

12-3

A function that can be described by $y = mx + b$ is a **linear function** and can be written $f(x) = mx + b$. You can use the Problem-Solving Guidelines to solve problems involving linear functions.

Solve.

13. A health club charges \$30 for a one-month membership plus \$1.50 for each aerobics class. Find the total cost for a month in which Adam attends 5 aerobics classes.

12-4

A function that can be described by $y = ax^2 + bx + c$ is a **quadratic function** and can be written $f(x) = ax^2 + bx + c$. If the domain of a quadratic function is the set of real numbers, then the graph of the function is a **parabola**. The x -coordinate of the **vertex**, the maximum or minimum point, is $-\frac{b}{2a}$.

Graph each function.

14. $f(x) = x^2 - 2$

15. $f(x) = \frac{1}{3}x^2$

12-5

An equation of the form $y = kx$, where k is constant, expresses **direct variation**. If one pair of values for x and y is known, substitute in the equation, $y = kx$, to find the value of k .

Find an equation of variation where y varies directly as x . One pair of values is given.

16. $y = 12$ when $x = 4$

17. $y = 4$ when $x = 8$

Solve.

18. A person's paycheck (P) varies directly as the number of hours (h) worked. The pay is \$165 for working 20 hours. Find the pay for 30 hours of work.

12-6

An equation of the form $y = \frac{k}{x}$, where k is constant, expresses **inverse variation**. If one pair of values for x and y is known, substitute in the equation to find k .

Find an equation of variation where y varies inversely as x . One pair of values is given.

19. $y = 5$ when $x = 6$

20. $y = 0.5$ when $x = 2$

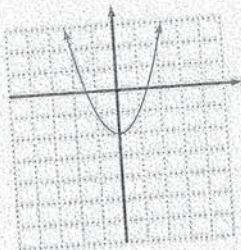
Solve.

21. It takes five hours for two washing machines to wash a fixed amount. How long would it take for ten washing machines to wash the same amount of clothes?

Chapter Wrap Up

13. $c(a) = 30 + 1.5a$; \$37.50

14.



15.



16. $y = 3x$

17. $y = \frac{1}{2}x$

18. \$247.50

19. $y = \frac{30}{x}$

20. $y = \frac{1}{x}$

21. 1 h

12-7

An equation of the form $z = kxy$, where k is a constant, expresses **joint variation**. An equation of the form $z = \frac{kx}{y}$, where k is a nonzero constant, expresses **combined variation**.

22. The area of a trapezoid varies jointly as its altitude and the sum of the bases. $A = ka(b_1 + b_2)$. If $A = 120$ when $a = 8$, $b_1 = 12$, and $b_2 = 18$,
- find the constant of variation.
 - find A when $a = 6$, $b_1 = 8$, and $b_2 = 12$.
23. W varies jointly as x and y and inversely as the square of z . If $W = 189$, $x = 28$, $y = 16$, and $z = 8$,
- find the constant of variation.
 - find W when $x = 24$, $y = 4$, and $z = 6$.

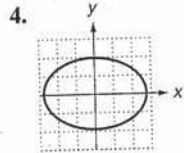
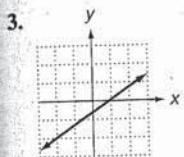
Assessment Item Analysis

Item	Lesson
1, 2	12-1
3, 4	12-2
5	12-3
6, 7	12-4
8	12-5
9, 10	12-6
11, 12	12-7

12 Chapter Assessment

- $f(x) = \frac{1}{2}x + 1$; find $f(0)$, $f(1)$, and $f(2)$
- $g(t) = -2|t| + 3$; find $g(-1)$, $g(0)$, and $g(3)$

Which of the following are graphs of functions?



5. Elena rents a car for \$19 per day plus \$0.20 per mile. Compute the cost of a 2-day trip of 310 miles.

Graph. State the domain and range.

- $f(x) = x^2 + 4x + 4$
- $f(x) = -x^2$
- Find an equation of variation where y varies directly as x , and $y = 6$ when $x = 3$.
- Find an equation of variation where y varies inversely with x and $y = 6$ when $x = 2$.
- It takes 3 hours for 2 cement mixers to mix a certain amount. How long would it take 5 cement mixers to do the job?
- y varies jointly with x and the square of z . One set of values is $y = 375$ when $x = 3$ and $z = 25$. Find y when $x = 8$ and $z = 6$.
- x varies jointly with y and z and inversely with m . One set of values is $x = 2$ when $m = 3$, $y = 4$, and $z = 6$. Find x when $y = 7.5$, $m = 3$, and $z = 8$.

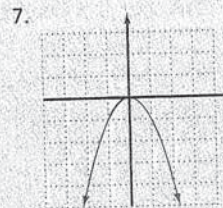
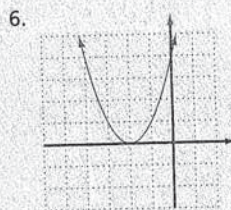
Chapter 12 Assessment 573

Chapter Wrap Up

- (a) $k = \frac{1}{2}$ (b) $A = 60$
- (a) $k = 27$ (b) $W = 72$

Chapter 12 Assessment

- $1; \frac{3}{2}; 2$
- $1; 3; -3$
- Yes
- No
- \$100



- $y = 2x$
- $y = \frac{12}{x}$
- $1\frac{1}{5}$ h
- $y = 0.2xz^2; 57.6$
- $x = \frac{yZ}{4m}; x = 5$