S
es 651–657

to preteach some of the key
this chapter. Particularly for
uage Learners (ELL),
e vocabulary before the
son begins gives students a
o understanding the new
ing new words on poster
g to the words as you say
splaying the poster for a
is a useful technique.

nsion (p. 285)
wo squares

rouping (p. 281)
pletely (p. 268)
nomials (p. 283)
p. 262)
ro products

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res (p. 270)

CHAPTER 6

What You'll Learn in Chapter 6

- How to factor special products and common factors
- How to factor trinomials
- How to use general strategies to factor polynomials
- How to solve polynomial equations by factoring

Skills & Concepts You Need for Chapter 6

1-5 Factor.

1. $6x + 6y$ **2.** $24w + 24z$ **3.** $4y + 28 + 12z$

3-4 Write as an algebraic expression.

4. 10 more than twice the number

5. 2 times the sum of a number and 6

3-5 Solve.

6. $6y + 4 = 2y + 8$ **7.** $3(2a + 4) = 20$

3-11

8. The perimeter of a rectangle is 280 cm. The length is 20 cm more than the width. Find the dimensions.

5-3 Multiply.

9. $(-6x^8)(2x^5)$ **10.** $(-6x^2y^2)(4xy^4)$

5-9 Multiply.

11. $9x(4x + 7)$ **12.** $3s(6t^4 - 2s^2 - 3t - 6)$

13. $(3x + 8)(x - 7)$ **14.** $(x + 3)(5x - 7)$

15. $8x(2x^2 - 6x + 1)$ **16.** $(x + 6)(x - 4)$

17. $(y - 8)(y + 3)$ **18.** $(7w + 6)(4w - 1)$

5-10 Multiply.

19. $(x - 9)^2$ **20.** $(5x + 3)^2$

21. $(a - 7)(a + 7)$ **22.** $(2 - 5y)(2 + 5y)$

Skills & Concepts You Need for Chapter 6

1. $6(x + y)$
2. $24(w + z)$
3. $4(y + 7 + 3z)$
4. $2n + 10$
5. $2(n + 6)$
6. 1
7. $\frac{4}{3}$
8. 80 cm, 60 cm
9. $-12x^{13}$
10. $-24x^3y^6$
11. $36x^2 + 63x$
12. $18st^4 - 6s^3 - 9st - 18s$
13. $3x^2 - 13x - 56$
14. $5x^2 + 8x - 21$
15. $16x^3 - 48x^2 + 8x$
16. $x^2 + 2x - 24$
17. $y^2 - 5y - 24$
18. $28w^2 + 17w - 6$
19. $x^2 - 18x + 81$
20. $25x^2 + 30x + 9$
21. $a^2 - 49$
22. $4 - 25y^2$

E/ASSESS

12x
+ 4)
$x^2y^6 + 20x^5y^3$
$- 2y^3 + 4x^3$)

Guide
exible scheduling, this
e split into parts.

1
52–60

eview to maintain skills.

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

6-1 Exercises

A

Find three factorizations for each monomial.

1. $6x^3$ **2.** $9y^4$ **3.** $-9a^5$
4. $-12x^6$ **5.** $24x^4y^2$ **6.** $15m^5n$
7. $-18p^3q^2$ **8.** $10r^2s^6$ **9.** $12a^3b^4$

Factor.

10. $x^2 - 4x$ **11.** $y^2 + 8y$ **12.** $2a^2 + 6a$
13. $3p^2 - 3p$ **14.** $3y^4 + 6y^2 + 6$ **15.** $5x^2 + 10x + 30$
16. $14m^4 - 12m$ **17.** $28y^2 + 21y^4$ **18.** $32x^5 - 17x^4$
19. $9x^3 + 25x^7$ **20.** $6a^2 - 5a^2$ **21.** $11y^4 + 7y^4$
22. $2x^2 + 2x - 8$ **23.** $6x^2 + 3x - 15$
24. $x^3y + 6x^2y$ **25.** $4a^4b^2 + a^2b$
26. $8x^4y^2 - 24x^2y$ **27.** $5m^5n + 10m^3$
28. $12m^5n^2 + 9m^4n + 6m^3n^2$ **29.** $2x^3y^2 + 6xy^3 + 8xy^2$
30. $17x^5 + 34x^3 + 51x$ **31.** $16x^6 - 32x^5 - 48x$
32. $6x^4 - 10x^3 + 3x^2$ **33.** $5x^5 + 10x^2 - 8x$
34. $x^5 + x^4 + x^3 - x^2$ **35.** $x^9 - x^7 + x^4 + x^3$
36. $2x^7 - 2x^6 - 64x^5 + 4x^3$ **37.** $10x^3 + 25x^2 + 15x - 20$
38. $4a^4b^4 - 2a^3b^2 + 6a^2$ **39.** $5p^3q^2 + 10p^2q^2 - 20pq^2$
40. $2x^3y - 4x^2y + x^2$ **41.** $6m^3n^3 + 3m^3n^2 + m^2n^2$

42. TEST PREP Which polynomial has $3x$ as a factor?
A. $15x^3 - 9x^2 + 13x$ **B.** $-21x^2 + 24y^2$
C. $9x - x^3$ **D.** $6x^2 + 15x$

B

Two polynomials are **relatively prime** if they have no common factors other than constants. Tell which pairs are relatively prime.

43. $5x, x^2$ **44.** $3x, ax - 3$
45. $x + x^2, 3x^3$ **46.** $y - 6, y$
47. $7a, a$ **48.** $2p^2 + 2, 2p$
49. $t^2 - 4t, t^2 - 4$ **50.** $3a^2 - a, a^3 - 2a$
51. $2x + 4, 2x^2 - 4$ **52.** $m^2 + 4mn, 3mn + 2m$

Exercises

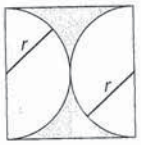
1–9. Answers may vary.
10. $x(x - 4)$
11. $y(y + 8)$
12. $2a(a + 3)$
13. $3p(p - 1)$
14. $3(y^4 + 2y^2 + 2)$
15. $5(x^2 + 2x + 6)$
16. $2m(7m^3 - 6)$
17. $7y^2(4 + 3y^2)$
18. $x^4(32x - 17)$
19. $x^3(9 + 25x^4)$
20. a^2
21. $18y^4$
22. $2(x^2 + x - 4)$
23. $3(2x^2 + x - 5)$
24. $x^2y(x + 6)$
25. $a^2b(4a^2b + 1)$
26. $8x^2y(x^2y - 3)$
27. $5m^3(m^2n + 2)$
28. $3m^2n(4m^2n + 3m + 2n)$
29. $2xy^2(x^2 + 3y + 4)$
30. $17x(x^4 + 2x^2 + 3)$
31. $16x(x^5 - 2x^4 - 3)$
32. $x^2(6x^2 - 10x + 3)$
33. $x(5x^4 + 10x - 8)$
34. $x^2(x^3 + x^2 + x - 1)$
35. $x^3(x^6 - x^4 + x + 1)$
36. $2x^3(x^4 - x^3 - 32x^2 + 2)$
37. $5(2x^3 + 5x^2 + 3x - 4)$
38. $2a^2(2a^2b^4 - ab^2 + 3)$
39. $5pq^2(p^2 + 2p - 4)$
40. $x^2(2xy - 4y + 1)$
41. $m^2n^2(6mn + 3m + 1)$
42. D
43. No
44. Yes
45. No
46. Yes

53. $4x^5 + 8x^3 - 6x, 8x^3 + 12x^2 + 24x - 16$

54. $6x^2y + 4xy + 2x, 2x^3 + 8x^2y + 14x$

55. $a^3 + a^2b + ab^2 + b^3, a^2 - ab^2$

56. ***Critical Thinking*** Represent the area (A) of the shaded region using an expression in factored form. (Formulas for area can be found in Table 2 in appendix.)



Quick Review

The area of a square with side s is s^2.

The area of a circle with radius r is πr^2.

Challenge

Find the common factor, if one exists.

57. $6t^3 + 30t^2, 9t^3 + 27t^2 + 9t$

58. $12t^4 - 15t^3 + 6t + 18, 16t^4 + 24t^3 - 48t^2 - 32t$

59. $192t^6 - 480t^4, 144t^8 + 72t^2$

60. $27x^5 - 81x^2 + 9x, 8x^4 - 16x + 4$

Mixed Review

Simplify. **61.** $x^7 \cdot x^5 \cdot x$ **62.** $(3m^2n^3)^2$ **63.** $(9t^2)(-2t^5)$ *5-1, 5-2, 5-3*

Write using scientific notation. **64.** 0.00437 **65.** 307.5 **66.** 5613 *5-4*

Collect like terms and then arrange in descending order for the variable m.

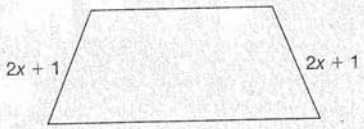
67. $12 - 9m^3 + 6m^2 + 4m^3 - 8m^2$ **68.** $5m^2 + 116 - 4m^2 + 3$ *5-6*

Connections: Geometry

Recall that the perimeter of a polygon is the sum of the lengths of its sides. Use the idea of perimeter to complete each of the following.

1. The perimeter of the parallelogram below is $8x + 4y$. What are the lengths of the missing sides?

2. The perimeter of the trapezoid below is $16x + 3$. The base of the trapezoid is 1 more than twice the length of the opposite side. What are the lengths of the missing sides?



47. No
48. Yes
49. Yes
50. No
51. Yes
52. No
53. Yes
54. No
55. Yes
56. $r^2(4 - \pi)$
57. $3t$
58. No common factor
59. $24t^2$
60. No common factor

Mixed Review

61. x^{13}
62. $9m^4n^6$
63. $-18t^7$
64. 4.37×10^{-3}
65. 3.075×10^2
66. 5.613×10^3
67. $-5m^3 - 2m^2 + 12$
68. $m^2 + 119$

Connections: Geometry

1. $3x + y$
   $3x + y$
2. shorter side: $4x$
   longer side: $8x + 1$

Teach the Mathematics (continued)

Chalkboard Examples

Factor.

1. $x^2 - 16$
   $(x + 4)(x - 4)$
2. $25x^2 - 4$
   $(5x)^2 - 2^2 = (5x + 2)(5x - 2)$
3. $25x^4 - 64y^2$
   $(5x^2)^2 - (8y)^2$
   $= (5x^2 + 8y)(5x^2 - 8y)$
4. $32x^2 - 50y^2$
   $2(16x^2 - 25y^2)$
   $= 2[(4x)^2 - (5y)^2]$
   $= 2(4x + 5y)(4x - 5y)$

Use Teaching Transparency T20 to present Chalkboard Example 1.

Factoring Completely

Show that factoring is the opposite of multiplication by factoring an expression and then multiplying the factors to return to the original expression.

Chalkboard Example

1. Factor.
   $a^4 - b^4$
   $(a^2)^2 - (b^2)^2$
   $= (a^2 + b^2)(a^2 - b^2)$
   $= (a^2 + b^2)(a + b)(a - b)$
   Note that $a^2 + b^2$ cannot be factored.

3. PRACTICE/ASSESS

LESSON QUIZ

Factor.

1. $a^2 - 16$
   $(a + 4)(a - 4)$
2. $25x^2 - 16$
   $(5x + 4)(5x - 4)$
3. $x^4 - 1$
   $(x^2 + 1)(x^2 - 1)$
   $= (x^2 + 1)(x + 1)(x - 1)$

Assignment Guide

To provide flexible scheduling, this lesson can be split into parts.

- Core 1–8
- Core 9–38, 47–69 odd
  Extension 71–74, 77, 79
- Core 39–46, 48–70 even, 75, 76
  Extension 78, 80–82

Use Mixed Review to maintain skills.

Try This

q. $(3x + 1)(3x - 1)(9x^2 + 1)$
r. $(4m^2 + n^4)(2m - n^2)(2m + n^2)$

Exercises

1. Yes
2. Yes
3. No
4. No
5. No
6. No
7. Yes
8. Yes
9. $(x + 2)(x - 2)$
10. $(x + 6)(x - 6)$
11. $(x + 3y)(x - 3y)$
12. $(m + y)(m - y)$
13. $(4a + 3)(4a - 3)$
14. $(5x + 2)(5x - 2)$
15. $(2x + 5)(2x - 5)$
16. $(3a + 4)(3a - 4)$
17. $(5m + 7)(5m - 7)$
18. $25(2x + 1)(2x - 1)$
19. $(x^2 + 3)(x^2 - 3)$
20. $(y^3 + 2)(y^3 - 2)$
21. $(m^8 + 5)(m^8 - 5)$
22. $(a^6 + 4)(a^3 + 2)(a^3 - 2)$
23. $4(x^2 + 2)(x^2 - 2)$
24. $(4x^3 + 5)(4x^3 - 5)$
25. $(8y^2 + 9)(8y^2 - 9)$
26. $(2x^5 + 5)(2x^5 - 5)$
27. $(6x^6 + 7)(6x^6 - 7)$
28. $(4y + 5)(4y - 5)$
29. $x(6 + 7x)(6 - 7x)$
30. $(11a^4 + 10)(11a^4 - 10)$
31. $y^2(9y^2 + 5)(9y^2 - 5)$
32. $y^4(10y + 7)(10y - 7)$
33. $2(2x + 7y)(2x - 7y)$
34. $6(2x + 3y^2)(2x - 3y^2)$
35. $2(4x + 5y)(4x - 5y)$
36. $3y^2(3 + 4y)(3 - 4y)$

PART 3 Factoring Completely

Objective: Factor a difference of two squares completely.

After you factor a difference of two squares, you can sometimes continue factoring. **Factoring completely** means to factor until factoring is no longer possible (other than for a common factor of 1).

EXAMPLE 10 Factor.

$$\begin{aligned} 1 - 16x^{12} &= (1)^2 - (4x^6)^2 \\ &= (1 - 4x^6)(1 + 4x^6) \\ &= [(1)^2 - (2x^3)^2](1 + 4x^6) \\ &= (1 + 2x^3)(1 - 2x^3)(1 + 4x^6) \end{aligned}$$

Factoring the first binomia as a difference of squares

Try This Factor.

q. $81x^4 - 1$ **r.** $16m^4 - n^8$

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

6-2 Exercises

A

Mental Math State whether each expression is a difference of two squares.

1. $x^2 - 4$ **2.** $x^2 - 36$ **3.** $x^2 + 36$ **4.** $x^2 + 4$
5. $x^2 - 35$ **6.** $x^2 - 50$ **7.** $-25 + 16x^2$ **8.** $-1 + 36x^2$

Factor.

9. $x^2 - 4$ **10.** $x^2 - 36$ **11.** $x^2 - 9y^2$
12. $m^2 - y^2$ **13.** $16a^2 - 9$ **14.** $25x^2 - 4$
15. $4x^2 - 25$ **16.** $9a^2 - 16$ **17.** $25m^2 - 49$
18. $100x^2 - 25$ **19.** $x^4 - 9$ **20.** $y^6 - 4$
21. $m^{16} - 25$ **22.** $-16 + a^{12}$ **23.** $-16 + 4x^4$
24. $16x^6 - 25$ **25.** $64y^4 - 81$ **26.** $4x^{10} - 25$
27. $36x^{12} - 49$ **28.** $16y^2 - 25$ **29.** $36x - 49x^3$
30. $121a^8 - 100$ **31.** $81y^6 - 25y^2$ **32.** $100y^6 - 49y^4$
33. $8x^2 - 98y^2$ **34.** $-54y^4 + 24x^2$ **35.** $-50y^2 + 32x^2$
36. $27y^2 - 48y^4$ **37.** $75m^6n^2 - 147$ **38.** $50a^{10}b^4 - 72$
39. $x^4 - 1$ **40.** $x^4 - 16$ **41.** $4x^4 - 64$ **42.** $5x^4 - 80$
43. $16 - y^4$ **44.** $25 - x^4$ **45.** $625 - m^4$ **46.** $4 - 9y^2$

B

Factor.

47. $16x - 81x^3$
48. $1 - y^8$
49. $b^8 - a^4$
50. $16x^2 - 25x^4$
51. $x^{16} - 9x^2$
52. $-16 + x^6$
53. $-81 + 49a^4$
54. $-64 + c^{14}$
55. $x^{12} - 16$
56. $^8 - 1$
57. $a^{12} - 4a^2$
58. $16p^8 - t^4$
59. $-9 + 25a^4$
60. $x^8 - 81$
61. $-49 + 9c^8$
62. $4x^4 - 4x^2$
63. $3x^5 - 12x^3$
64. $3x^2 - \frac{1}{3}$
65. $18x^3 - \frac{8}{25}x$
66. $x^2 - 2.25$
67. $x^3 - \frac{x}{16}$
68. $3.24x^2 - 0.81$
69. $0.64x^2 - 1.21$
70. $1.28x^2 - 2$
71. $(x + 3)^2 - 9$
72. $(y - 5)^2 - 36$
73. $(3a + 4)^2 - 49$
74. $(2y - 7)^2 - 1$
75. $y^8 - 256$
76. $x^{16} - 1$
77. ***Critical Thinking*** Can you find a rational nonzero value for b that allows you to factor $x^2 + b^2$? Explain.

Challenge

78. Find 2 polynomials, each with three factors, where the only common factor is $(x + 2)$.
79. Find a polynomial with 2 factors where one factor is $(x^2 - 2)$.
80. Find a third-degree polynomial where one factor is $(x + 5)$ and the other 2 factors are binomials.
81. Find a third-degree polynomial where there are three binomial factors and one factor is $(a + 2b)$.
82. Find a fourth-degree polynomial where there are three factors and one factor is $x^2 - 5$.

Mixed Review

Multiply. **83.** $(x + 7)(x + 7)$ **84.** $(2a - 3)(2a - 3)$ *5-10*

Identify the degree of each term and the degree of the polynomial.

85. $12c + 1$ **86.** $6y^3 - 25y^2 - 8y + 15$ **87.** $9a^2c^3 + 45ac - 7$ *5-5*

Write each as a decimal. **88.** 20% **89.** 8% **90.** 430% **91.** 6.5% *3-10*

Solve. **92.** $26 - 3c = 10c$ **93.** $4t - 12 = t + 3$ *3-5*

94. Ana has saved \$68 to buy the bicycle at the right. How much more money does Ana need? *3-4, 3-11*

95. Find the length of the base of a triangle if one side is 3 cm longer than the base, and the other side is 5 cm shorter than the base. The perimeter of the triangle is 52 cm. *3-4, 3-11*

Exercises

37. $3(5m^3n + 7)(5m^3n - 7)$
38. $2(5a^5b^2 + 6)(5a^5b^2 - 6)$
39. $(x^2 + 1)(x + 1)(x - 1)$
40. $(x^2 + 4)(x + 2)(x - 2)$
41. $4(x^2 + 4)(x + 2)(x - 2)$
42. $5(x^2 + 4)(x + 2)(x - 2)$
43. $(4 + y^2)(2 + y)(2 - y)$
44. $(5 + x^2)(5 - x^2)$
45. $(25 + m^2)(5 + m)(5 - m)$
46. $(2 + 3y)(2 - 3y)$
47. $x(4 + 9x)(4 - 9x)$
48. $(1 + y^4)(1 + y^2)(1 + y)(1 - y)$
49. $(b^4 + a^2)(b^2 + a)(b^2 - a)$
50. $x^2(4 + 5x)(4 - 5x)$
51. $x^2(x^7 + 3)(x^7 - 3)$
52. $(x^3 + 4)(x^3 - 4)$
53. $(7a^2 + 9)(7a^2 - 9)$
54. $(c^7 + 8)(c^7 - 8)$
55. $(x^6 + 4)(x^3 + 2)(x^3 - 2)$
56. $(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$
57. $a^2(a^5 + 2)(a^5 - 2)$
58. $(4p^4 + t^2)(2p^2 + t)(2p^2 - t)$
59. $(5a^2 + 3)(5a^2 - 3)$
60. $(x^4 + 9)(x^2 + 3)(x^2 - 3)$
61. $(3c^4 + 7)(3c^4 - 7)$
62. $4x^2(x + 1)(x - 1)$
63. $3x^3(x + 2)(x - 2)$
64. $3\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right)$
65. $2x\left(3x + \frac{2}{5}\right)\left(3x - \frac{2}{5}\right)$
66. $(x + 1.5)(x - 1.5)$
67. $x\left(x + \frac{1}{4}\right)\left(x - \frac{1}{4}\right)$
68. $(1.8x + 0.9)(1.8x - 0.9)$
69. $(0.8x + 1.1)(0.8x - 1.1)$
70. $2(0.8x + 1)(0.8x - 1)$
71. $x(x + 6)$
72. $(y + 1)(y - 11)$
73. $3(a - 1)(3a + 11)$
74. $4(y - 4)(y - 3)$
75. $(y^4 + 16)(y^2 + 4)(y +$
76. $(x^8 + 1)(x^4 + 1)(x^2 +$
$(x + 1)(x - 1)$
77. No; any product of the
$(x + a)(x + c)$ has 3 t
$x^2 + (a + c)x + ac$. F
have 2 terms, a and c
opposites; but if a and
then ac must be nega
cannot equal b^2 for a
Answers may vary. The ar
examples.
78. $x(x + 2)(x + 1) = x^3$
$(x + 2)(x + 3)(x - 3$
$x^3 + 2x^2 - 9x - 18$
79. $(x^2 - 2)(x^2 + 2) = x$
80. $(x + 5)(x + 1)(x - 1$
$= x^3 + 5x^2 - x - 5$
81. $(a + 2b)(a + b)(a -$
$= a^3 - ab^2 + 2a^2b -$
82. $(x^2 - 5)(x + 1)(x -$

Mixed Review

83. $x^2 + 14x + 49$
84. $4a^2 - 12a + 9$
85. 1, 0; 1
86. 3, 2, 1, 0; 3
87. 5, 2, 0; 5
88. 0.2
89. 0.08
90. 4.3
91. 0.065
92. 2
93. 5
94. \$132
95. The base is 18 cm.

2 Factoring Trinomial Squares

Objective: Factor trinomial squares.

To factor trinomial squares, you can use the following relationships.

$$A^2 + 2AB + B^2 = (A + B)^2$$
$$A^2 - 2AB + B^2 = (A - B)^2$$

Remember to factor out a common factor first, if possible.

EXAMPLES Factor.

4 $x^2 + 6x + 9 = x^2 + 2 \cdot x \cdot 3 + 3^2 = (x + 3)^2$ The sign of the middle term is positive.

5 $x^2 - 14x + 49 = x^2 - 2 \cdot x \cdot 7 + 7^2 = (x - 7)^2$ The sign of the middle term is negative.

6 $16a^2 - 40ab + 25b^2 = (4a - 5b)^2$

7 $27m^2 + 72mn + 48n^2 = 3(9m^2 + 24mn + 16n^2)$ Factoring out the common factor, 3.

$= 3(3m + 4n)^2$

Try This Factor.

g. $x^2 + 2x + 1$ **h.** $x^2 - 2x + 1$

i. $25x^2 - 70x + 49$ **j.** $48m^2 + 120mn + 75n^2$

6-3 Exercises

A

Mental Math Which of the following are trinomial squares?

1. $x^2 - 14x + 49$ **2.** $x^2 - 16x + 64$ **3.** $x^2 + 16x - 64$

4. $x^2 - 14x - 49$ **5.** $x^2 - 6x + 9$ **6.** $x^2 + 2x + 4$

7. $8x^2 + 40x + 25$ **8.** $9x^2 + 18xy + 9y^2$

9. $36m^2 - 24m + 16n^2$ **10.** $16x^2 - 56xy + 49y^2$

Factor. Remember to look first for a common factor.

11. $x^2 - 14x + 49$ **12.** $x^2 - 16x + 64$ **13.** $x^2 + 16x + 64$

14. $x^2 + 14x + 49$ **15.** $x^2 - 2x + 1$ **16.** $x^2 + 2x + 1$

17. $x^2 + 4xy + 4y^2$ **18.** $x^2 - 4xy + 4y^2$ **19.** $y^2 - 6xy + 9x^2$

20. $y^2 + 6xy + 9x^2$ **21.** $2x^2 - 4x + 2$ **22.** $2x^2 - 40x + 200$

23. $x^3 - 18x^2 + 81x$ **24.** $x^3 + 24x^2 + 144x$

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

Try This

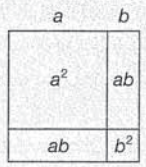
g. $(x + 1)^2$
h. $(x - 1)^2$
i. $(5x - 7)^2$
j. $3(4m + 5n)^2$

Exercises

1. Yes
2. Yes
3. No
4. No
5. Yes
6. No
7. No
8. Yes
9. No
10. Yes
11. $(x - 7)^2$
12. $(x - 8)^2$
13. $(x + 8)^2$
14. $(x + 7)^2$
15. $(x - 1)^2$
16. $(x + 1)^2$
17. $(x + 2y)^2$
18. $(x - 2y)^2$
19. $(y - 3x)^2$
20. $(y + 3x)^2$
21. $2(x - 1)^2$
22. $2(x - 10)^2$
23. $x(x - 9)^2$
24. $x(x + 12)^2$

2 Factoring Trinomial Sq

Remind students that the follov model illustrates that
$(a + b)^2 = a^2 + 2ab + b^2$



Have students substitute $-b$ int equation

$(a + (-b))^2 = a^2 + 2a(-b)$
$= a^2 - 2ab +$

Then have them multiply $(a -$ to get the same result. Remind to check their factorization usin multiplication.

Chalkboard Examples

Factor.

1. $x^2 + 10x + 25$
   $x^2 + 2 \cdot x \cdot 5 + 5^2 = (x + 5$
2. $y^2 - 8y + 16$
   $y^2 - 2 \cdot y \cdot 4 + 4^2 = (y - 4$
3. $4x^2 + 12x + 9$
   $(2x)^2 + 2 \cdot 2x \cdot 3 + 3^2 = (2$
4. $2x^2 + 12x + 18$
   $2(x^2 + 6x + 9) = 2(x + 3)^2$

Use Teaching Transparency T20 Chalkboard Example 1.

LESSON ENRICHMENT

The expressions

$x^3 + 9x^2 + 27x + 27$ $(x +$
$x^3 + 12x^2 + 48x + 64$ $(x +$
$x^3 + 15x^2 + 75x + 125$ $(x +$

are all cubes of binomials.
Determine what each binomial i
Determine a pattern for recogni cubes of binomials.

3. PRACTICE/ASSESS

LESSON QUIZ

Factor.

1. $x^2 + 20x + 100$ $(x + 10)^2$
2. $y^2 - 4y + 4$ $(y - 2)^2$
3. $4a^2 + 12ab + 9b^2$ $(2a + 3b)$
4. $7x^3 + 14x^2 + 7x$
   $7x(x^2 + 2x + 1) = 7x(x + 1)$

California Assessment Prep

Choose the best answer.

1. Find the greatest common factor.
$15m^7n^5 + 21m^4n^6 - 36m^8n^4$

A $3m^4n^4$
B $5m^3n + 7n^2 - 12m^4$
C $3mn$
D 1

2. Factor, if possible.
$16q^2 - 8q - 1$

F $(4q - 1)(4q - 1)$
G $(4q - 1)(4q + 1)$
H $(16q - 1)(q + 1)$
J It cannot be factored using only integers.

25. $20x^2 + 100x + 125$
26. $12x^2 + 36xy + 27y^2$
27. $49y^2 - 42xy + 9x^2$
28. $64y^2 - 112xy + 49x^2$
29. $5y^4 + 10y^2 + 5$
30. $a^4 + 14a^2 + 49$
31. $y^6 + 26y^3 + 169$
32. $y^6 + 16y^3 + 64$
33. $16x^{10} - 8x^5 + 1$
34. $9x^{10} + 12x^5 + 4$
35. $4x^4 + 4x^2 + 1$
36. $1 + 2a^4 + a^8$
37. $81x^6 + 72x^3y + 16y^2$
38. $9a^8 - 30a^4b + 25b^2$

B

Factor, if possible.

39. $49x^2 - 216$
40. $27x^3 - 13x$
41. $x^2 + 22x + 121$
42. $4x^2 + 9$
43. $x^2 - 5x + 25$
44. $18x^3 + 12x^2 + 2x$
45. $63x - 28$
46. $162x^2 - 82x$
47. $x^4y^4 - 9y^4$
48. $81x^2 - 64x$
49. $x^8 - 2^8$
50. $3^4 - x^4$

Factor.

51. $(y + 3)^2 + 2(y + 3) + 1$
52. $(a + 4)^2 - 2(a + 4) + 1$
53. $4(a + 5)^2 + 20(a + 5) + 25$
54. $49(x + 1)^2 - 42(x + 1) + 9$
55. $(x + 7)^2 - 4x - 24$
56. $(a + 4)^2 - 6a - 15$

57. ***Critical Thinking*** Suppose $x^2 + a^2x + a^2$ factors into $(x + a)^2$. Find the nonzero value of a.

Challenge

Factor.

58. $9x^{18} + 48x^9 + 64$
59. $x^{2n} + 10x^n + 25$

Factor as the square of a binomial, then as a difference of two squares.

60. $a^2 + 2a + 1 - 9$
61. $y^2 + 6y + 9 - x^2 - 8x - 16$

Find c so that the polynomial will be the square of a binomial.

62. $cy^2 + 6y + 1$
63. $cy^2 - 24y + 9$

64. ***Mathematical Reasoning*** Show that the difference of the squares of two consecutive integers is the sum of the integers. (Hint: Use x for the smaller number.)

Mixed Review

Multiply. **65.** $(9 - x^2)(1 + 2x)$ **66.** $(y^3 + y^2)(y^3 - 4)$ *5-9*

Factor. **67.** $6x^2 - 9x$ **68.** $24a^2 - 12a$ **69.** $9y^3 + 3y$ *6-1*

Write using standard notation. **70.** 1.667×10^{-3} **71.** 3.594×10^5 *5-4*

1. A; Algebra 11.0
2. J; Algebra 11.0

37. $(9x^3 + 4y)^2$
38. $(3a^4 - 5b)^2$
39. Not possible
40. $x(27x^2 - 13)$
41. $(x + 11)^2$
42. Not possible
43. Not possible
44. $2x(3x + 1)^2$
45. $7(9x - 4)$
46. $2x(81x - 41)$
47. $y^4(x^2 - 3)(x^2 + 3)$
48. $x(81x - 64)$
49. $(x^4 + 16)(x^2 + 4)(x + 2)(x - 2)$
50. $(9 + x^2)(3 + x)(3 - x)$
51. $(y + 4)^2$
52. $(a + 3)^2$
53. $(2a + 15)^2$
54. $(7x + 4)^2$
55. $(x + 5)^2$
56. $(a + 1)^2$
57. $a = 2$
58. $(3x^9 + 8)^2$
59. $(x^n + 5)^2$
60. $(a + 1)^2 - 3^2 = (a + 4)(a - 2)$
61. $(y + 3)^2 - (x + 4)^2 = (y + x + 7)(y - x - 1)$
62. 9
63. 16
64. $(x + 1)^2 - x^2 = 2x + 1$

Mixed Review

65. $-2x^3 - x^2 + 18x + 9$
66. $y^6 + y^5 - 4y^3 - 4y^2$
67. $3x(2x - 3)$
68. $12a(2a - 1)$
69. $3y(3y^2 + 1)$
70. 0.001667
71. 359,400

PRACTICE/ASSESS

ON QUIZ

or.

$^2 + 8x + 7$
$x + 1)(x + 7)$
$^2 - 8a + 15$
$a - 3)(a - 5)$
$^2 - 5xy - 14y^2$
$x + 2y)(x - 7y)$

Assignment Guide

To provide flexible scheduling, this lesson can be split into parts.

Core 1–21, 37, 38, 40, 42–44
Extension 47, 48, 54, 57
Core 22–36, 39, 41, 45, 46
Extension 49–53, 55, 56

Use Mixed Review to maintain skills.

EXAMPLE 5 Factor $a^2 + ab - 6b^2$.

We are looking for binomials of the form $(a \_\_ b)(a \_\_ b)$. Find two numbers whose sum is 1 and whose product is -6.

Product of -6		Sum
1,	-6	-5
-1,	6	5
2,	-3	-1
-2,	3	1 ← The numbers we need are -2 and 3.

$a^2 + ab - 6b^2 = (a - 2b)(a + 3b)$

Try This Factor.

i. $x^2 + 4x - 12$ **j.** $x^2 - 4x - 12$
k. $a^2 + 5ab - 14b^2$ **l.** $x^2 - xy - 30y^2$

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

6-4 Exercises

A

Factor.

1. $x^2 + 8x + 15$
2. $x^2 + 5x + 6$
3. $x^2 + 7x + 12$
4. $x^2 + 9x + 8$
5. $x^2 - 6x + 9$
6. $y^2 + 11y + 28$
7. $x^2 + 9x + 14$
8. $a^2 + 11a + 30$
9. $b^2 + 5b + 4$
10. $x^2 - 11x + 28$
11. $a^2 - 14a + 48$
12. $z^2 - 8z + 7$
13. $m^2 + 10m + 21$
14. $a^2 - 14a + 45$
15. $z^2 - 10z + 24$
16. $t^2 + 12tp + 36p^2$
17. $a^2 - 9ab + 20b^2$
18. $x^2 - 5xy + 4y^2$
19. $c^2 - 7cd + 10d^2$
20. $x^2 - 8xy + 15y^2$
21. $y^2 - 11yz + 10z^2$
22. $x^2 - 2x - 15$
23. $x^2 + x - 42$
24. $x^2 + 2x - 15$
25. $x^2 - 7x - 18$
26. $y^2 - 3y - 28$
27. $x^2 - 6x - 16$
28. $x^2 - x - 42$
29. $y^2 - 4y - 45$
30. $x^2 - 7x - 60$
31. $x^2 - 2xy - 99y^2$
32. $x^2 + 6xy - 72y^2$
33. $c^2 + cd - 56d^2$
34. $b^2 + 5bc - 24c^2$
35. $a^2 + 2ab - 35b^2$
36. $y^2 - xy - 2x^2$

Try This

i. $(x + 6)(x - 2)$
j. $(x - 6)(x + 2)$
k. $(a + 7b)(a - 2b)$
l. $(x - 6y)(x + 5y)$

Exercises

1. $(x + 5)(x + 3)$
2. $(x + 3)(x + 2)$
3. $(x + 4)(x + 3)$
4. $(x + 8)(x + 1)$
5. $(x - 3)^2$
6. $(y + 7)(y + 4)$
7. $(x + 7)(x + 2)$
8. $(a + 6)(a + 5)$
9. $(b + 4)(b + 1)$
10. $(x - 7)(x - 4)$
11. $(a - 8)(a - 6)$
12. $(z - 1)(z - 7)$
13. $(m + 7)(m + 3)$
14. $(a - 9)(a - 5)$
15. $(z - 4)(z - 6)$
16. $(t + 6p)^2$
17. $(a - 5b)(a - 4b)$
18. $(x - 4y)(x - y)$
19. $(c - 5d)(c - 2d)$
20. $(x - 5y)(x - 3y)$
21. $(y - 10z)(y - z)$
22. $(x - 5)(x + 3)$
23. $(x + 7)(x - 6)$
24. $(x + 5)(x - 3)$
25. $(x - 9)(x + 2)$
26. $(y - 7)(y + 4)$
27. $(x - 8)(x + 2)$
28. $(x - 7)(x + 6)$
29. $(y - 9)(y + 5)$
30. $(x - 12)(x + 5)$
31. $(x - 11y)(x + 9y)$
32. $(x + 12y)(x - 6y)$
33. $(c + 8d)(c - 7d)$
34. $(b + 8c)(b - 3c)$
35. $(a + 7b)(a - 5b)$
36. $(y + x)(y - 2x)$

B

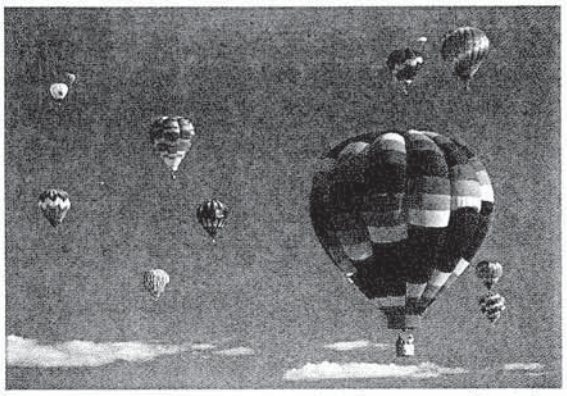
Factor.

37. $x^2 + 20x + 100$
38. $x^2 + 20x + 99$
39. $x^2 - 21x - 100$
40. $x^2 - 20x + 96$
41. $x^2 - 21x - 72$
42. $4x^2 + 40x + 100$
43. $x^2 - 25x + 144$
44. $y^2 - 21y + 108$
45. $a^2 + a - 132$
46. $a^2 + 9a - 90$
47. $120y^2 - 23xy + x^2$
48. $96e^2 + 22de + d^2$
49. $108y^2 - 3xy - x^2$
50. $112z^2 + 9yz - y^2$

51. ***Critical Thinking*** A hot air balloon flies at a speed of $(n + 8)$ miles per hour. At this rate, how long will it take to fly $(n^2 + 5n - 24)$ miles?

52. ***Mathematical Reasoning***
Let $x^2 - 3x - 10 = (x + a)(x + b)$.
   a. What do you know about the signs of a and b?
   b. Suppose $|a| > |b|$. Which number, a or b, is a negative integer? How do you know?

53. ***Mathematical Reasoning***
Let $x^2 + 3x - 10 = (x + a)(x + b)$.
   a. What do you know about the signs of a and b?
   b. Suppose $|a| > |b|$. Which number, a or b, is a negative integer? How do you know?



Hot air balloons move with wind currents. Evaluate the expressions in Exercise 51 when $n = 27$ to find the speed and length of one journey.

Challenge

54. Find all integers m for which $y^2 + my + 50$ can be factored.
55. Find all integers b for which $a^2 + ba - 50$ can be factored.

Factor completely.

56. $x^3 - x^2 - 6x$
57. $-x^3 + 22x^2 + 23x$

Mixed Review

Find three factorizations for each monomial. **58.** $12x^3$ **59.** $8y^3$ *6-1*

Simplify. **60.** $(3m + 14) - (m + 9)$ **61.** $\frac{a^6b^3}{a^2b^2}$ **62.** $\frac{6m^6}{3m^6}$ *2-8, 5-2*

63. $(4y^2 - 3) + (y^2 + 11)$ **64.** $\frac{5t^3}{11t}$ **65.** $\frac{4c^3}{4c^2}$ *5-3, 5-7, 5-8*

Solve. **66.** $|x| + 3 = 7$ **67.** $3|c| - 2 = 4$ **68.** $4 - |m| = -2$ *3-8*

Exercises

37. $(x + 10)^2$
38. $(x + 11)(x + 9)$
39. $(x - 25)(x + 4)$
40. $(x - 12)(x - 8)$
41. $(x - 24)(x + 3)$
42. $4(x + 5)^2$
43. $(x - 16)(x - 9)$
44. $(y - 12)(y - 9)$
45. $(a + 12)(a - 11)$
46. $(a + 15)(a - 6)$
47. $(15y - x)(8y - x)$
48. $(16e + d)(6e + d)$
49. $(12y + x)(9y - x)$
50. $(7z + y)(16z - y)$
51. $(n - 3)$ hours

Photo caption: 35 miles/hr, 840 miles

52. a. The signs of a and b must be opposite.
   b. Since the middle term is negative, the number with the larger absolute value must be negative. Therefore, a must be a negative integer.
53. a. The signs of a and b must be opposite.
   b. Since the middle term is positive, the number with the larger absolute value must be positive. Therefore, b is a negative integer.
54. $-51, -27, -15, 15, 27, 51$
55. $-49, -23, -5, 5, 23, 49$
56. $x(x - 3)(x + 2)$
57. $-x(x - 23)(x + 1)$

Mixed Review

Answers may vary for 58 and 59.

58. $(12x)(x^2)$, $(6x^2)(2x)$, $(4x)(3x^2)$,
59. $8y(y^2)$, $(4y^2)(2y)$, $(2y^2)(4y)$, etc.
60. $2m + 5$
61. a^4b
62. 2
63. $5y^2 + 8$
64. $\frac{5t^2}{11}$
65. c
66. 4, −4
67. 2, −2
68. 6, −6

EXAMPLES

2 Factor $2x^2 + 5x - 12$.

First terms: Find two numbers whose product is 2.
Last terms: Find two numbers whose product is -12.

$(2x + 3)(x - 4)$ $(2x - 2)(x + 6)$ $(2x - 1)(x + 12)$ Possible
$(2x - 3)(x + 4)$ $(2x + 2)(x - 6)$ $(2x - 12)(x + 1)$ factorizations

The outside product plus the inside product must equal $5x$.

$$2x^2 + 5x - 12 = (2x - 3)(x + 4)$$

3 Factor $8m^2 + 8m - 6$.

$8m^2 + 8m - 6 = 2(4m^2 + 4m - 3)$ Factoring out the common factor, 2

First terms: Find two numbers whose product is 4.
Last terms: Find two numbers whose product is -3.

$(4m + 3)(m - 1)$ $(4m - 3)(m + 1)$ $(2m + 3)(2m - 1)$ Possible
$(4m - 1)(m + 3)$ $(4m + 1)(m - 3)$ $(2m - 3)(2m + 1)$ factorizations

The outside product plus the inside product must equal $4m$.

$$8m^2 + 8m - 6 = 2(4m^2 + 4m - 3) = 2(2m + 3)(2m - 1)$$

Try This Factor.

d. $3x^2 - 21x + 36$ e. $8x^2 - 2$ f. $9a^2 - 15a - 6$
g. $2x^2 + 4x - 6$ h. $4a^2 + 2a - 6$ i. $6m^2 + 15mn - 9n^2$

6-5 Exercises

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A

Factor.

1. $2x^2 - 7x - 4$
2. $3x^2 - x - 4$
3. $5x^2 + x - 18$
4. $3x^2 - 4x - 15$
5. $6x^2 + 23x + 7$
6. $6x^2 + 13x + 6$
7. $3x^2 + 4x + 1$
8. $7x^2 + 15x + 2$
9. $4x^2 + 4x - 15$
10. $9a^2 + 6a - 8$
11. $2x^2 - x - 1$
12. $15n^2 - 19n - 10$
13. $9x^2 + 18x - 16$
14. $2y^2 + 5y + 2$
15. $3x^2 - 5x - 2$
16. $18c^2 - 3c - 10$
17. $12x^2 + 31x + 20$
18. $15x^2 + 19x - 10$
19. $14x^2 + 19x - 3$
20. $35x^2 + 34x + 8$
21. $9p^2 + 18p + 8$
22. $6 - 13x + 6x^2$
23. $49 - 42b + 9b^2$
24. $15x^2 - 19x + 6$
25. $24x^2 + 47x - 2$
26. $16a^2 + 78a + 27$
27. $35x^2 - 57x - 44$
28. $9a^2 + 12a - 5$
29. $20 + 6x - 2x^2$
30. $15 + x - 2x^2$

Try This

d. $3(x - 3)(x - 4)$
e. $2(2x + 1)(2x - 1)$
f. $3(3a + 1)(a - 2)$
g. $2(x + 3)(x - 1)$
h. $2(2a + 3)(a - 1)$
i. $3(2m - n)(m + 3n)$

Exercises

1. $(2x + 1)(x - 4)$
2. $(3x - 4)(x + 1)$
3. $(5x - 9)(x + 2)$
4. $(3x + 5)(x - 3)$
5. $(2x + 7)(3x + 1)$
6. $(2x + 3)(3x + 2)$
7. $(3x + 1)(x + 1)$
8. $(7x + 1)(x + 2)$
9. $(2x + 5)(2x - 3)$
10. $(3a - 2)(3a + 4)$
11. $(2x + 1)(x - 1)$
12. $(3n - 5)(5n + 2)$
13. $(3x + 8)(3x - 2)$
14. $(2y + 1)(y + 2)$
15. $(3x + 1)(x - 2)$
16. $(6c - 5)(3c + 2)$
17. $(3x + 4)(4x + 5)$
18. $(3x + 5)(5x - 2)$
19. $(7x - 1)(2x + 3)$
20. $(7x + 4)(5x + 2)$
21. $(3p + 4)(3p + 2)$
22. $(2 - 3x)(3 - 2x)$
23. $(7 - 3b)^2$
24. $(3x - 2)(5x - 3)$
25. $(x + 2)(24x - 1)$
26. $(8a + 3)(2a + 9)$
27. $(7x + 4)(5x - 11)$
28. $(3a - 1)(3a + 5)$
29. $2(5 - x)(2 + x)$
30. $(5 + 2x)(3 - x)$

3. Factor $18x^2 + 36x - 14$.
Factor out the common fact
$2(9x^2 + 18x - 7)$
This factors into
$2(3x - 1)(3x + 7)$.

3. PRACTICE/ASSESS

LESSON QUIZ

1. Factor $2x^2 + 7x + 3$.
$(2x + 1)(x + 3)$
2. Factor $6x^2 + 17x + 5$.
$(3x + 1)(2x + 5)$

Assignment Guide
Core 1–60
Extension 61–69

Use Mixed Review to maint

Factor.

31. $12x^2 + 28x - 24$
32. $6c^2 - 33c + 15$
33. $30x^2 - 24x - 54$
34. $20x^2 - 25x + 5$
35. $6x^2 + 4x - 10$
36. $18y^2 - 21y - 9$
37. $3a^2 - 4a + 1$
38. $6x^2 + 13x + 6$
39. $12x^2 - 28x - 24$
40. $6x^2 + 33x + 15$
41. $2x^2 + x - 1$
42. $15s^2 + 19s + 6$
43. $9b^2 - 18b - 16$
44. $14x^2 + 35x + 14$
45. $15x^2 - 25x - 10$
46. $30b^2 - b - 20$

B

Factor, if possible.

47. $18x^2 + 3xy - 10y^2$
48. $12a^2 - 31ab + 20b^2$
49. $15m^2 - 19mn - 10n^2$
50. $14p^2 - 19pq - 3q^2$
51. $35x^2 - 34xy + 8y^2$
52. $56a^2 - 15ab + b^2$
53. $9x^4 + 18x^2 + 8$
54. $6y^2 - 13y + 6$
55. $9x^2 - 42x + 49$
56. $15x^4 - 19x^2 + 6$
57. $6a^3 + 4a^2 - 10a$
58. $18x^3 - 21x^2 - 9x$
59. $x^2 + 3x - 7$
60. $b^2 + 13b - 12$
61. $x^5 + x^3 - 6x$
62. $x^5 - 6x^3 + 5x$

63. ***Critical Thinking*** What are the values of a and c in the trinomial square $ax^2 + 12x + c$ if $a \neq 1$ and $c > a$?

64. ***Error Analysis*** Bobbi factored $4y^2 + 36y + 80$ as $2(y + 5)(y + 4)$. She argued that using the distributive property, $2(y + 5) = 2y + 10$ and $2(y + 4) = 2y + 8$. We know that $(2y + 10)(2y + 8) = 4y^2 + 36y + 80$. Thus $2(y + 5)(y + 4) = 4y^2 + 36y + 80$. Write a paragraph evaluating Bobbi's argument.

Challenge

Factor.

65. $20x^{2n} + 16x^n + 3$
66. $-15x^{2m} + 26x^m - 8$
67. $x^{6a} - x^{3a} - 6$
68. $x^{4n+1} + 2x^{2n+1} + x$
69. $3(a + 1)^{n+1}(a + 3)^2 - 5(a + 1)^n(a + 3)^3$

Mixed Review

Write as an algebraic expression. **70.** 11 more than the product of m and n **71.** the sum of a and b, divided by 2 **72.** the square of the difference between x and y **73.** r divided by the sum of s and t *1-6, 3-4*

Solve. **74.** $3 - 4y < 7$ **75.** $6t > 9 + 9t$ **76.** $8 - 3y \leq 2$ *4-4*

47. $(6x + 5y)(3x - 2y)$
48. $(3a - 4b)(4a - 5b)$
49. $(5m + 2n)(3m - 5n)$
50. $(7p + q)(2p - 3q)$
51. $(5x - 2y)(7x - 4y)$
52. $(8a - b)(7a - b)$
53. $(3x^2 + 2)(3x^2 + 4)$
54. $(3 - 2y)(2 - 3y)$
55. $(3x - 7)^2$
56. $(5x^2 - 3)(3x^2 - 2)$
57. $2a(3a + 5)(a - 1)$
58. $3x(2x - 3)(3x + 1)$
59. Not factorable
60. Not factorable
61. $x(x^2 + 3)(x^2 - 2)$
62. $x(x^2 - 5)(x + 1)(x - 1)$
63. $a = 4, c = 9$
64. Answers may vary, but students should recognize that Bobbi should have factored 4 out of the polynomial.
$(2y + 10)(2y + 8)$
$= 2(y + 5) \cdot 2(y + 4)$
$= 4(y + 5)(y + 4)$
65. $(10x^n + 3)(2x^n + 1)$
66. $(-3x^m + 4)(5x^m - 2)$
67. $(x^{3a} - 3)(x^{3a} + 2)$
68. $x(x^{2n} + 1)^2$
69. $-2(a + 1)^n(a + 3)^2(a + 6)$

Mixed Review

70. $mn + 11$
71. $\frac{a + b}{2}$
72. $(x - y)^2$
73. $\frac{r}{s + t}$
74. $y > -1$
75. $t < -3$
76. $y \geq 2$

CA 11.0: Apply basic factoring techniques to third degree polynomials.

Factoring by Grouping

6-6

Objective: Factor polynomials by grouping.

What You'll Learn

1. To factor polynomials by grouping

. . . And Why

To factor polynomials with more than three terms

The distributive property can be used to factor some polynomials with four terms. Consider $x^3 + x^2 + 2x + 2$.

There is no factor common to all terms other than 1. We can, however, factor $x^3 + x^2$ and $2x + 2$ separately.

$$x^3 + x^2 = x^2(x + 1) \qquad 2x + 2 = 2(x + 1)$$

Therefore, $x^3 + x^2 + 2x + 2 = x^2(x + 1) + 2(x + 1)$. We can use the distributive property again and factor out the common factor, $x + 1$.

$$x^2(x + 1) + 2(x + 1) = (x + 1)(x^2 + 2)$$

This method is called **factoring by grouping.** Not all expressions with four terms can be factored by this method.

EXAMPLES Factor.

1 $6x^3 - 9x^2 + 4x - 6 = (6x^3 - 9x^2) + (4x - 6)$

$= 3x^2(2x - 3) + 2(2x - 3)$ Factoring each binomial

$= (2x - 3)(3x^2 + 2)$ Factoring out the common factor, $2x - 3$

2 $x^3 + x^2 + x + 1 = (x^3 + x^2) + (x + 1)$

$= x^2(x + 1) + 1(x + 1)$ Factoring each binomial

$= (x + 1)(x^2 + 1)$ Factoring out the common factor, $x + 1$

3 $x^3 + 2x^2 - x - 2 = (x^3 + 2x^2) + (-x - 2)$

$= x^2(x + 2) + 1(-x - 2)$

$= x^2(x + 2) - 1(x + 2)$ Using $ab = (-a)(-b)$

$= (x + 2)(x^2 - 1)$

$= (x + 2)(x + 1)(x - 1)$ Factoring completely

4 $x^2y^2 + ay^2 + ab + bx^2 = y^2(x^2 + a) + b(x^2 + a)$

$= (x^2 + a)(y^2 + b)$

5 $x^3 + x^2 + 2x - 2 = x^2(x + 1) + 2(x - 1)$

This cannot be factored by grouping.

Try This Factor.

a. $8x^3 + 2x^2 + 12x + 3$

b. $4x^3 - 6x^2 - 6x + 9$

c. $x^3 + x^2 - x - 1$

d. $3a - 6b + 5a^2 - 10ab$

Try This

a. $(4x + 1)(2x^2 + 3)$
b. $(2x - 3)(2x^2 - 3)$
c. $(x + 1)(x + 1)(x - 1)$
d. $(a - 2b)(3 + 5a)$

CA: 11.0

6-6

TIME-FOCUS METER

1. FOCUS

FIRST FIVE MINUTES

1. Factor.
   $3x^2 + 5x + 2$
   $(3x + 2)(x + 1)$
2. Simplify.
   $x(x^2 + x) + 2(x + 1)$
   $x^3 + x^2 + 2x + 2$

2. TEACH the Mathematics

You may want to show that the grouping can be done in more than one way. For example, an alternative way to work Example 4 would be to factor x^2 in the first and fourth terms and a in the second and third terms.

$x^2y^2 + ay^2 + ab + bx^2$
$= x^2(y^2 + b) + a(y^2 + b)$
$= (x^2 + a)(y^2 + b)$

The final factorization is the same.

Point out that $(a + b) = -(-a - b)$ and that $(a - b) = -(-a + b) = -(b - a)$.

Key Questions

- Does $(7 - x) = -(x - 7)$?
  Yes
- Does $(y - 2) = -(2 + y)$?
  No

Chalkboard Examples

Factor.

1. $2x^3 + 6x^2 + x + 3$
   $2x^2(x + 3) + 1(x + 3)$
   $= (2x^2 + 1)(x + 3)$
2. $4a^3 + 10a^2 + 6a + 15$
   $2a^2(2a + 5) + 3(2a + 5)$
   $= (2a^2 + 3)(2a + 5)$
3. $5x^4 - 5x^3 - x + 1$
   $5x^3(x - 1) - 1(x - 1)$
   $= (5x^3 - 1)(x - 1)$
4. $2x^3 + 2x^2 - 8x - 8$
   $2(x^3 + x^2 - 4x - 4)$
   $= 2(x^3 - 4x + x^2 - 4)$
   $= 2[x(x^2 - 4) + 1(x^2 - 4)]$
   $= 2(x + 1)(x^2 - 4)$
   $= 2(x + 1)(x + 2)(x - 2)$
5. $4x^3 + 3x^2 + 2x + 1$
   $x^2(4x + 3) + 2x + 1$
   The method doesn't work in this case

CTICE/ASSESS

QUIZ

$x^2 + x + 1$
$- 1) + (x + 1)$
$- 1)(x + 1)$
$6x^2 + 2x + 4$
$+ 2) + 2(x + 2)$
$^2 + 2)(x + 2)$

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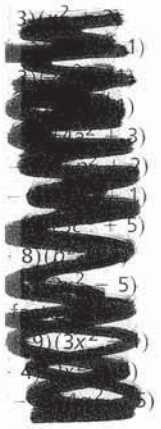
lixed Review to maintain skills.

$(x+3)(x^2+2)$
$(2z+1)(3z^2+1)$
$(y+3)(2y^2+1)$
$(3x+2)(x^2+1)$
$(2a-3)(4a^2+3)$
$(2p-5)(5p^2+2)$
$(3x-4)(4x^2+1)$
$(6c-7)(3c^2+5)$
$(b+8)(b^2-3)$
$(x+6)(2x^2-5)$
not Factorable
$(8x+9)(3x^2-1)$
$(x-4)(2x^2-9)$
$(5g-1)(4g^2-5)$

es



Practice Multiple Choice

Choose the best answer.

1. Which statement is true about factoring $x^2 + 5x + 6$?

A In factored form, each x term has a coefficient of 1.
B It is a "difference of squares" problem.
C It is a "trinomial square" problem.
D $x^2 + 5x + 6$ cannot be factored.

2. Factor completely. $24x^2 + 82x + 70$

F $2(2x + 5)(6x + 7)$
G $3(8x^2 + 27x + 23)$
H $2(3x + 5)(4x + 7)$
J It cannot be factored using only rational numbers.

1. A; Algebra 11.0
2. H; Algebra 11.0

6-6 Exercises

A

Factor.

1. $x^3 + 3x^2 + 2x + 6$
2. $6z^3 + 3z^2 + 2z + 1$
3. $2y^3 + 6y^2 + y + 3$
4. $3x^3 + 2x^2 + 3x + 2$
5. $8a^3 - 12a^2 + 6a - 9$
6. $10p^3 - 25p^2 + 4p - 10$
7. $12x^3 - 16x^2 + 3x - 4$
8. $18c^3 - 21c^2 + 30c - 35$
9. $b^3 + 8b^2 - 3b - 24$
10. $2x^3 + 12x^2 - 5x - 30$
11. $14x^3 + 18x^2 - 21x + 27$
12. $24x^3 + 27x^2 - 8x - 9$
13. $2x^3 - 8x^2 - 9x + 36$
14. $20g^3 - 4g^2 - 25g + 5$
15. $ax - bx + ay - by$
16. $bx + 2b + cx + 2c$
17. $n^2 + 2n + np + 2p$
18. $2x^2 - 4x + xz - 2z$
19. $a^2 - 3a + ay - 3y$
20. $6y^2 - 3y + 2py - p$

B

Factor.

21. $4x^5 + 6x^3 + 6x^2 + 9$
22. $4y^5 + 6y^4 + 6y^3 + 9y^2$
23. $c^6 - c^4 - c^2 + 1$
24. $x^{13} + x^7 + 2x^6 + 2$

Factor each as a difference of two squares.

25. $(x - y)^2 - z^2$
26. $4 - (2a + 3b)^2$
27. $a^2 + 2ab + b^2 - 1$
28. $c^2 - 6cd + 9d^2 - 4$

29. ***Critical Thinking*** What is the relationship between the value of D in the polynomial and the values of a, b, and c in the binomials?

$$Ax^3 + Bx^2 + Cx + D = (x + a)(x + b)(x + c)$$

Challenge

30. ***Mathematical Reasoning*** Factor $acx^{m+n} + adx^n + bcx^m + bd$ into two factors. Assume a, b, c, and d are constants. Verify your factorization by multiplying.

31. Find $ax^3 + bx^2 + cx + d$ so that a, b, c, and d are integers, $\frac{a}{c} = \frac{b}{d} = 4$, and $\frac{a}{b} = \frac{7}{5}$. Factor the result by grouping.

32. Subtract $(x^2 + 1)^2$ from $x^2(x + 1)^2$ and factor the result.

Mixed Review

Multiply. **33.** $(m + n)^2$ **34.** $(m - n)^2$ **35.** $(m + n)(m - n)$ *5-10*

Factor. **36.** $x^2 - 16$ **37.** $y^2 + 6y + 9$ **38.** $3a^2 - 6a + 3$

39. $c^2 - c - 90$ **40.** $n^2 - 15n + 54$ **41.** $20 - 4x - 5y + xy$

42. $x^2 + 3x + 2$ **43.** $9a^4 - b^2$ **44.** $25 - 40a + 16a^2$ *6-2, 6-3, 6-4, 6-5*

15. $(a - b)(x + y)$
16. $(b + c)(x + 2)$
17. $(n + p)(n + 2)$
18. $(2x + z)(x - 2)$
19. $(a + y)(a - 3)$
20. $(3y + p)(2y - 1)$
21. $(2x^2 + 3)(2x^3 + 3)$
22. $y^2(2y + 3)(2y^2 + 3)$
23. $(c^2 + 1)(c + 1)(c + 1)(c - 1)(c - 1)$
24. $(x^7 + 2)(x^6 + 1)$
25. $(x - y + z)(x - y - z)$
26. $(2 + 2a + 3b)(2 - 2a - 3b)$
27. $(a + b + 1)(a + b - 1)$
28. $(c - 3d + 2)(c - 3d - 2)$
29. $D = abc$
30. $(ax^n + b)(cx^m + d)$
31. $28x^3 + 20x^2 + 7x + 5$, $(7x + 5)(4x^2 + 1)$
32. $x^2(x + 1)^2 - (x^2 + 1)^2 = x^4 + 2x^3 + x^2 - (x^4 + 2x^2 + 1) = 2x^3 - 2x^2 + x^2 - 1 = 2x^2(x - 1) + (x + 1)(x - 1) = (2x^2 + x + 1)(x - 1)$

Mixed Review

33. $m^2 + 2mn + n^2$
34. $m^2 - 2mn + n^2$
35. $m^2 - n^2$
36. $(x + 4)(x - 4)$
37. $(y + 3)(y + 3)$ or $(y + 3)^2$
38. $3(a - 1)(a - 1)$ or $3(a - 1)^2$
39. $(c + 9)(c - 10)$
40. $(n - 6)(n - 9)$
41. $(5 - x)(4 - y)$
42. $(x + 1)(x + 2)$
43. $(3a^2 + b)(3a^2 - b)$
44. $(5 - 4a)^2$

CA 11.0: Apply basic factoring techniques including common factors, the difference of squares, and perfect squares of binomials.

...toring: A General Strategy

6-7

Objective: Factor polynomials.

...re is a good strategy for factoring.

...oring Polynomials

Always look first for a common factor.
Then look at the number of terms.
Two terms: Determine whether you have a difference of squares.
Three terms: Determine whether the trinomial is a square of a binomial. If not, test the factors of the terms.
Four terms: Try factoring by grouping.
Always factor completely.

EXAMPLES Factor.

1 $10x^3 - 40x$

A. Look first for a common factor.

$10x^3 - 40x = 10x(x^2 - 4)$ Factoring out the greatest common factor

B. Factor a difference of two squares.

$10x(x + 2)(x - 2)$ Factoring $x^2 - 4$

C. Have we factored completely? Yes, because no factor can be factored further.

2 $t^4 - 16 = (t^2 + 4)(t^2 - 4)$ Factoring a difference of two squares

$= (t^2 + 4)(t + 2)(t - 2)$ Factoring a difference of two squares again

3 $2a^3 + 10a^2 + a + 5 = (2a^3 + 10a^2) + (a + 5)$

$= 2a^2(a + 5) + 1(a + 5)$ Factoring each binomial

$= (2a^2 + 1)(a + 5)$ Using the distributive property

4 $x^4 - 10x^2 + 25 = (x^2)^2 - 10x^2 + 25$ Writing an equivalent expression

$= (x^2 - 5)^2$ Factoring a trinomial square

Try This Factor.

a. $3m^4 - 3$ b. $x^6 + 8x^3 + 16$ c. $2x^4 + 8x^3 + 6x^2$
d. $3x^3 + 12x^2 - 2x - 8$ e. $8x^3 - 200x$ f. $y^5 - 2y^4 - 35y^3$

What You'll Learn

1. To factor polynomials

. . . And Why

To be able to factor polynomials completely

Try This

a. $3(m^2 + 1)(m + 1)(m - 1)$
b. $(x^3 + 4)^2$
c. $2x^2(x + 3)(x + 1)$
d. $(x + 4)(3x^2 - 2)$
e. $8x(x + 5)(x - 5)$
f. $y^3(y - 7)(y + 5)$

CA: 11.0

6-7

TIME-FOCUS METER

1. FOCUS

FIRST FIVE MINUTES

Factor.

1. $3x^3 + 6x^2$
   $3x^2(x + 2)$
2. $x^2 - 64$
   $(x - 8)(x + 8)$
3. $x^2 + 2x + 1$
   $(x + 1)^2$
4. $x^2 + 4x + 3$
   $(x + 1)(x + 3)$

2. TEACH the Mathemati

Emphasize that a factorization can be easily checked by multiplication.

Key Questions

Are these products correct?

- $(x + 3)(x - 3) = x^2 - 3$
  No, $x^2 - 9$
- $(x + 6)^2 = x^2 + 8x + 36$
  No, $x^2 + 12x + 36$
- $(x + 3)(x + 5) = x^2 + 8x + 15$
  Yes

Math Point

In general, polynomials of the form $x^{\text{odd}} + 1$, $x^{\text{odd}} - 1$, or $x^{\text{even}} - 1$ can be factored over the reals. This will be taught in future courses.

Chalkboard Examples

Factor.

1. $14a^4 - 14a^2$
   $14a^2(a^2 - 1)$
   $= 14a^2(a + 1)(a - 1)$
2. $x^8 - 1$
   $(x^4)^2 - 1$
   $= (x^4 + 1)(x^4 - 1)$
   $= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
   $= (x^4 + 1)(x^2 + 1)(x + 1)(x -$
3. $12x^3 - 21x^2 + 8x - 14$
   Factor by grouping, since there four terms.
   $3x^2(4x - 7) + 2(4x - 7)$
   $= (3x^2 + 2)(4x - 7)$

$0x^3 + 75x^2$
$^2 + 10x + 25)$
binomial square.
$5)^2$
$4a^3 + 24a^2$
$- 7a + 12)$
$+ 3)(a + 4)$

TICE/ASSESS

UIZ

$7x$
$9) = 3x(x + 3)(x - 3)$
$x^2 + 36x$
$12x + 36) = x(x + 6)^2$
$a^2 + 3a$
$7a + 3)$
$+ 1)(a + 3)$

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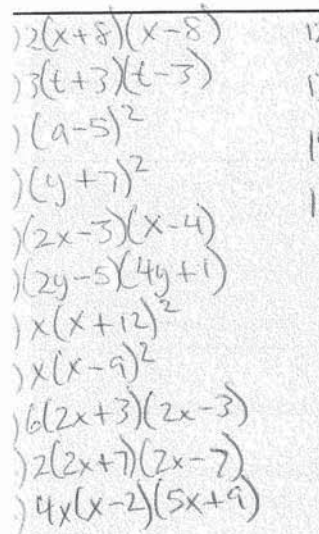
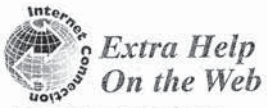
ed Review to maintain skills.





Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

6-7 Exercises

A

Factor.

1. $2x^2 - 128$
2. $3t^2 - 27$
3. $a^2 + 25 - 10a$
4. $y^2 + 49 + 14y$
5. $2x^2 - 11x + 12$
6. $8y^2 - 18y - 5$
7. $x^3 + 24x^2 + 144x$
8. $x^3 - 18x^2 + 81x$
9. $24x^2 - 54$
10. $8x^2 - 98$
11. $20x^3 - 4x^2 - 72x$
12. $9x^3 + 12x^2 - 45x$
13. $x^2 + 4$
14. $t^2 + 25$
15. $x^5 - 14x^4 + 49x^3$
16. $2x^6 + 8x^5 + 8x^4$
17. $x^2 + 3x + 1$
18. $x^2 + 5x + 2$
19. $4x^4 - 64$
20. $5x^5 - 80x$
21. $1 - y^8$
22. $t^8 - 1$
23. $x^5 - 4x^4 + 3x^3$
24. $x^6 - 2x^5 + 7x^4$
25. $x^3 + 3x^2 - 4x - 12$
26. $x^3 - 5x^2 - 25x + 125$
27. $x^4 + 7x^2 - 3x^3 - 21x$
28. $m^4 + 8m^3 + 8m^2 + 64m$

B

Factor completely.

29. $a^4 - 2a^2 + 1$
30. $x^4 + 9$
31. $20 - 6x - 2x^2$
32. $45 - 3x - 6x^2$
33. $18 + y^3 - 9y - 2y^2$
34. $-(x^4 - 7x^2 - 18)$
35. $a^3 + 4a^2 + a + 4$
36. $x^3 + x^2 - (4x + 4)$
37. $12a^3b^2 - 6a^2b + 4a^2b^2 - 2ab$
38. $a^3 - 5a^2b - 14ab^2$
39. $3x^3y - 2x^2y^2 + 3x^4y - 2x^3y^2$
40. $m^2n^2 + 7mn^3 + 10n^4$
41. ***Critical Thinking*** The polynomials $x^4 + 3x^2 - 28$ and $x^2 + 7x + 10$ have a common binomial factor. What is it?

Challenge

42. Factor $64a^4 + 1$. (Hint: Write it as $64a^4 + 16a^2 + 1 - 16a^2$.)
43. Factor $x^{2h} - 2^{2h}$ when $h = 4$.

Mixed Review

Express using positive exponents. **44.** m^{-2} **45.** x^{-1} **46.** $3c^{-2}$ *5-1*

Subtract. **47.** $(8a^5 + a^3 - 1) - (2a^5 + 4a^3 - 1)$

48. $(-3a^2b + 7ab - 4a) - (-2a^2b - 4a + 3ab)$ *5-8*

12) $3x(x+3)(3x-5)$
13) Not Factorable
14) Not Factorable
15) $x^3(x-7)^2$

16. $2x^4(x + 2)^2$
17. Not factorable
18. Not factorable
19. $4(x^2 + 4)(x + 2)(x - 2)$
20. $5x(x^2 + 4)(x + 2)(x - 2)$
21. $(y^4 + 1)(y^2 + 1)(y + 1)(1 - y)$
22. $(t^4 + 1)(t^2 + 1)(t + 1)(t - 1)$
23. $x^3(x - 3)(x - 1)$
24. $x^4(x^2 - 2x + 7)$
25. $(x - 2)(x + 2)(x + 3)$
26. $(x - 5)^2(x + 5)$
27. $x(x - 3)(x^2 + 7)$
28. $m(m^2 + 8)(m + 8)$
29. $(a + 1)(a + 1)(a - 1)(a - 1)$
30. Not factorable
31. $-2(x - 2)(x + 5)$ or $2(2 - x)(5 + x)$
32. $-3(2x - 5)(x + 3)$ or $3(5 - 2x)(3 + x)$
33. $(y - 2)(y + 3)(y - 3)$
34. $-1(x^2 + 2)(x + 3)(x - 3)$
35. $(a + 4)(a^2 + 1)$
36. $(x + 2)(x - 2)(x + 1)$
37. $2ab(2ab - 1)(3a + 1)$
38. $a(a - 7b)(a + 2b)$
39. $x^2y(x + 1)(3x - 2y)$
40. $n^2(m + 5n)(m + 2n)$
41. $x + 2$
42. $(8a^2 - 4a + 1)(8a^2 + 4a + 1)$
43. $(x^4 + 16)(x^2 + 4)(x + 2)(x - 2)$

Mixed Review

44. $\frac{1}{m^2}$
45. $\frac{1}{x}$
46. $\frac{3}{c^2}$
47. $6a^5 - 3a^3$
48. $-a^2b + 4ab$

Connections: Discrete Math

Binomial Expansion

The distributive property can be used as many times as needed to expand binomials to powers. This is called a **binomial expansion.**

$$\begin{aligned}(x + 2)^3 = (x + 2)(x + 2)(x + 2) &= (x + 2)(x^2 + 4x + 4)\\ &= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8\\ &= x^3 + 6x^2 + 12x + 8\end{aligned}$$

Consider the following expansions of $(a + b)^n$ for $n = 0, 1, 2, 3, 4,$ and 5. Look for patterns.

$$\begin{aligned}(a + b)^0 &= 1\\ (a + b)^1 &= a + b\\ (a + b)^2 &= a^2 + 2ab + b^2\\ (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3\\ (a + b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\\ (a + b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5\end{aligned}$$

Compare the above expansions with *Pascal's Triangle.*

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
```

We can use Pascal's Triangle and the patterns in the above expansions to find powers of binomials.

EXAMPLE Expand $(x + 3)^4$.

We can use the pattern shown above.

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

We substitute x for a and 3 for b.

$$\begin{aligned}(x + 3)^4 &= x^4 + 4 \cdot x^3 \cdot 3 + 6 \cdot x^2 \cdot 3^2 + 4 \cdot x \cdot 3^3 + 3^4\\ &= x^4 + 4 \cdot x^3 \cdot 3 + 6 \cdot x^2 \cdot 9 + 4 \cdot x \cdot 27 + 81\\ &= x^4 + 12x^3 + 54x^2 + 108x + 81\end{aligned}$$

Exercises

Use the patterns above to expand the binomials.

1. $(m + n)^5$
2. $(x + 1)^3$
3. $(a + 4)^4$
4. Write the next row of numbers in Pascal's Triangle.
5. Use Pascal's Triangle to find $(a + b)^6$.

Exercises

1. $m^5 + 5m^4n + 10m^3n^2 + 10m^2n^3 + 5mn^4 + n^5$
2. $x^3 + 3x^2 + 3x + 1$
3. $a^4 + 16a^3 + 96a^2 + 256a + 256$
4. 1 6 15 20 15 6 1
5. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Connections: Discrete Math

Use Teaching Transparency T22 f
Triangle.

Math Point

The binomial expansion pat
known as Pascal's Triangle h
history stretching back cent
before Blaise Pascal was bor
1623. References indicate th
was known to the Persian p
and mathematician Omar
Khayyam, who lived in the e
12th century, and that it wa
known in China at this time
When Chu Shih-Chieh publi
the triangle in 1303 in Chin
referred to it as an "old" m

Teach the Mathematics (continued)

$x^2 + 10x + 25 = 0$.

r.

$5)^2 = 0$

$5 = 0$ or $x + 5 = 0$

$x = -5$ or $x = -5$

e is only one solution in this case,

e $9x^2 - 4 = 0$.

$+ 2)(3x - 2) = 0$

$+ 2 = 0$ or $3x - 2 = 0$

$3x = -2$ or $3x = 2$

$x = -\frac{2}{3}$ or $x = \frac{2}{3}$

)N ENRICHMENT

s a nonstandard method for finding lution to a second-degree equation. $x^2 - x - 1 = 0$. Rewrite the tion as

$^2 = x + 1$

e both sides by x, if x is not 0.

$= 1 + \frac{1}{x}$

ıave solved for x in terms of x. titute for x in the denominator.

$x = 1 + \frac{1}{1 + \frac{1}{x}}$

can substitute again, and again.

$x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ldots}}}$

lculate the expression shown above. ıes it give an approximate solution to e equation? Yes

Check:

$x^2 + 5x = -6$	
$(-2)^2 + 5(-2)$	-6
$4 + (-10)$	-6
-6	-6 ✔

$x^2 + 5x = -6$	
$(-3)^2 + 5(-3)$	-6
$9 + (-15)$	-6
-6	-6 ✔

The solutions are -2 and -3.

Try This Solve.

e. $x^2 - x - 6 = 0$ **f.** $m^2 - m = 56$ **g.** $x^2 - 3x = 28$

Example 4 shows that the value of the polynomial $x^2 + 5x + 6$ is 0 when $x = -2$ or $x = -3$. For a polynomial containing a variable, any value of the variable that makes the value of the polynomial equal 0 is called a zero or a **root** of the polynomial.

EXAMPLE 5 Find the roots of the polynomial $x^2 - 5x$.

$x^2 - 5x = 0$ Setting the polynomial equal to 0

$x(x - 5) = 0$ Factoring

$x = 0$ or $x - 5 = 0$

$x = 0$ or $x = 5$

Check:

$x^2 - 5x = 0$	
$0^2 - 5(0)$	0
0	0 ✔

$x^2 - 5x = 0$	
$5^2 - 5(5)$	0
0	0 ✔

The roots of $x^2 - 5x$ are 0 and 5.

EXAMPLE 6 Find the roots of $4x^2 - 25$.

$4x^2 - 25 = 0$ Setting the polynomial equal to 0

$(2x - 5)(2x + 5) = 0$ Factoring a difference of two squares

$2x - 5 = 0$ or $2x + 5 = 0$

$2x = 5$ or $2x = -5$

$x = \frac{5}{2}$ or $x = -\frac{5}{2}$

Check:

$4x^2 - 25 = 0$	
$4\left(\frac{5}{2}\right)^2 - 25$	0
$4\left(\frac{25}{4}\right) - 25$	0
0	0 ✔

$4x^2 - 25 = 0$	
$4\left(-\frac{5}{2}\right)^2 - 25$	0
$4\left(\frac{25}{4}\right) - 25$	0
0	0 ✔

The roots are $\frac{5}{2}$ and $-\frac{5}{2}$.

Try This Find the roots of each polynomial.

h. $x^2 + 6x + 9$ **i.** $x^2 + 4x$ **j.** $25x^2 - 16$

Try This

e. 3, −2
f. 8, −7
g. 7, −4
h. −3
i. 0, −4
j. $\frac{4}{5}$, $-\frac{4}{5}$

Exercises

1. −8, −6
2. −3, −2
3. 3, −5
4. −9, 3
5. −12, 11
6. 13, −53
7. 0, −5
8. 0, −7
9. 0, 13
10. 0, 4
11. 0, −10
12. 0, 21
13. 4, $\frac{1}{4}$
14. $\frac{11}{12}$, $\frac{5}{8}$
15. 0, $\frac{2}{3}$
16. 0, $\frac{9}{8}$

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6-8 Exercises

A

Solve.

1. $(x + 8)(x + 6) = 0$
2. $(c + 3)(c + 2) = 0$
3. $(a - 3)(a + 5) = 0$
4. $(x + 9)(x - 3) = 0$
5. $(x + 12)(x - 11) = 0$
6. $(x - 13)(x + 53) = 0$
7. $x(x + 5) = 0$
8. $y(y + 7) = 0$
9. $y(y - 13) = 0$
10. $v(v - 4) = 0$
11. $0 = y(y + 10)$
12. $0 = x(x - 21)$
13. $(7x - 28)(28x - 7) = 0$
14. $(12x - 11)(8x - 5) = 0$
15. $2x(3x - 2) = 0$
16. $75x(8x - 9) = 0$
17. $\frac{1}{2}x\left(\frac{2}{3}x - 12\right) = 0$
18. $\frac{5}{7}d\left(\frac{3}{4}d - 6\right) = 0$
19. $\left(\frac{1}{3} - 3x\right)\left(\frac{1}{5} - 2x\right) = 0$
20. $\left(\frac{1}{5} + 2x\right)\left(\frac{1}{9} - 3x\right) = 0$
21. $\left(\frac{1}{3}y - \frac{2}{5}\right)\left(\frac{1}{4}y - \frac{3}{2}\right) = 0$
22. $\left(\frac{7}{4}x - \frac{1}{12}\right)\left(\frac{2}{3}x - \frac{12}{11}\right) = 0$
23. $(0.3x - 0.1)(0.05x - 1) = 0$
24. $(0.1x - 0.3)(0.4x - 20) = 0$
25. $9x(3x - 2)(2x - 1) = 0$
26. $(x - 5)(x + 55)(5x - 1) = 0$
27. $x^2 + 6x + 5 = 0$
28. $x^2 + 7x - 18 = 0$
29. $x^2 + 4x - 21 = 0$
30. $b^2 - 8b + 15 = 0$
31. $x^2 - 8x = 0$
32. $x^2 - 3x = 0$
33. $x^2 + 19x = 0$
34. $x^2 - 100 = 0$
35. $9x^2 - 4 = 0$
36. $4a^2 - 9 = 0$
37. $x^2 + 6x + 9 = 0$
38. $x^2 + 10x + 25 = 0$
39. $12y^2 - 5y = 2$
40. $2y^2 + 12y = -10$
41. $x(x - 5) = 14$
42. $t(3t + 1) = 2$
43. $64m^2 = 81$
44. $100t^2 = 49$
45. $(4x + 9)(14x - 7) = 0$
46. $(3w - 1)(w + 2) = 0$
47. $5x^2 = 6x$
48. $(5x + 1)(4x - 12) = 0$
49. $x^2 - 2x + 1 = 0$
50. $(3x - 9)(x + 3) = 0$
51. $6x^2 - 4x = 10$
52. $(2x + 5)(x + 4) = 0$
53. $(2x + 9)(x + 8) = 0$
54. $v^2 - 6v - 16 = 0$

Exercises

17. 0, 18
18. 0, 8
19. $\frac{1}{9}, \frac{1}{10}$
20. $-\frac{1}{10}, \frac{1}{27}$
21. 2, 6
22. $\frac{1}{21}, \frac{18}{11}$
23. $\frac{1}{2}$, 20
24. 3, 50
25. 0, $\frac{2}{3}, \frac{1}{2}$
26. 5, $-55, \frac{1}{5}$
27. $-5, -1$
28. 2, -9
29. -7, 3
30. 5, 3
31. 0, 8
32. 0, 3
33. 0, -19
34. 10, -10
35. $\frac{2}{3}, -\frac{2}{3}$
36. $\frac{3}{2}, -\frac{3}{2}$
37. -3
38. -5
39. $\frac{2}{3}, -\frac{1}{4}$
40. $-5, -1$
41. 7, -2
42. $\frac{2}{3}, -1$
43. $\frac{9}{8}, -\frac{9}{8}$
44. $\frac{7}{10}, -\frac{7}{10}$
45. $-\frac{9}{2}, \frac{1}{2}$
46. $\frac{1}{3}, -2$
47. 0, $\frac{6}{5}$
48. 3, $-\frac{1}{5}$
49. 1
50. 3, -3
51. $\frac{5}{3}, -1$
52. $-\frac{5}{2}, -4$
53. $-\frac{9}{2}, -8$
54. 8, -2

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

3. PRACTICE/ASSESS

LESSON QUIZ

Solve.

1. $(x - 3)(x + 5) = 0$
   $x = 3$ or $x = -5$
2. $(2x - 1)x = 0$
   $x = \frac{1}{2}$ or $x = 0$
3. $x^2 + 4x + 3 = 0$
   $x = -1$ or $x = -3$
4. $x^2 + 6x + 9 = 0$
   $x = -3$

Assignment Guide

To provide flexible scheduling lesson can be split into parts.

- Core 1–26
  Extension 70
- Core 27–69
  Extension 71, 72

Use Mixed Review to maintain

Exercises

1. $-8, -6$
2. $-3, -2$
3. 3, -5
4. -9, 3
5. -12, 11
6. 13, -53
7. 0, -5
8. 0, -7
9. 0, 13
10. 0, 4
11. 0, -10
12. 0, 21
13. $4, \frac{1}{4}$
14. $\frac{11}{12}, \frac{5}{8}$
15. $0, \frac{2}{3}$
16. $0, \frac{9}{8}$

Find the roots of each polynomial.

55. $c^2 - 16$ **56.** $d^2 + 7d + 6$ **57.** $x^2 - 9x + 14$

58. $x^2 + 12x$ **59.** $3x^2 - 7x - 20$ **60.** $7x^2 - 8x$

B

Solve.

61. $b(b + 9) = 4(5 + 2b)$ **62.** $y(y + 8) = 16(y - 1)$

63. $(t - 3)^2 = 36$ **64.** $(t - 5)^2 = 2(5 - t)$

65. $x^2 - \frac{1}{64} = 0$ **66.** $x^2 - \frac{25}{36} = 0$

67. $\frac{5}{16}x^2 = 5$ **68.** $\frac{27}{25}x^2 = \frac{1}{3}$

69. ***Critical Thinking*** Write all factorable second-degree trinomials whose first term is x^2 and whose last term is -16.

70. **TEST PREP** Which equation does *not* have –1 as a solution?

A. $(x + 1)(x - 2) = 0$ **B.** $(x + 3)(x - 1) = 0$

C. $x^2 - x = -2x$ **D.** $(x + 3)(x + 1) = 0$

Challenge

71. Find an equation that has the given numbers as solutions. For example, 3 and -2 are solutions to $x^2 - x - 6 = 0$.

a. 1, −3 **b.** 3, −1 **c.** 2, 2 **d.** 3, 4 **e.** 3, −4

f. −3, 4 **g.** −3, −4 **h.** $\frac{1}{2}, \frac{1}{2}$ **i.** 5, −5 **j.** $0, 0.1, \frac{1}{4}$

72. For each equation in the left-hand column, find an equation in the right-hand column that has the same two solutions.

a. $3x^2 - 4x + 8 = 0$	**(1)** $4x^2 + 8x + 36 = 0$
b. $(x - 6)(x + 3) = 0$	**(2)** $(2x + 8)(2x - 5) = 0$
c. $x^2 + 2x + 9 = 0$	**(3)** $9x^2 - 12x + 24 = 0$
d. $(2x - 5)(x + 4) = 0$	**(4)** $(x + 1)(5x - 5) = 0$
e. $5x^2 - 5 = 0$	**(5)** $x^2 - 3x - 18 = 0$
f. $x^2 + 10x - 2 = 0$	**(6)** $2x^2 + 20x - 4 = 0$

Mixed Review

Write as an algebraic expression. **73.** 3 times the quantity a number plus 8 **74.** 3 times the sum of a number and 8 **75.** half of the difference of a number and 15 **76.** the square of the sum of a and b **77.** the sum of the squares of a and b **78.** the difference of the squares of a and b *1-6, 3-4*

Factor. **79.** $4y^2 + 10y - 6$ **80.** $4x^2 - y^2$ **81.** $6y^2 - 5y - 6$ *6-2, 6-5*

55.) 4, −4
56.) −6, −1
57.) 7, 2
58) 0, −12
59) 4, $-\frac{5}{3}$
60) 0, $\frac{8}{7}$
61) 4, −5
62) 4
63) 9, −3
64) 5, 3
65) $\frac{1}{8}$, $-\frac{1}{8}$
66) $\frac{5}{6}$, $-\frac{5}{6}$

67. 4, −4

68. $\frac{5}{9}, \frac{-5}{9}$

69. $x^2 - 15x - 16$, $x^2 + 15x - 16$, $x^2 - 6x - 16$, $x^2 + 6x - 16$

70. B

71. a. $x^2 + 2x - 3 = 0$
b. $x^2 - 2x - 3 = 0$
c. $x^2 - 4x + 4 = 0$
d. $x^2 - 7x + 12 = 0$
e. $x^2 + x - 12 = 0$
f. $x^2 - x - 12 = 0$
g. $x^2 + 7x + 12 = 0$
h. $x^2 - x + \frac{1}{4} = 0$ or $4x^2 - 4x + 1 = 0$
i. $x^2 - 25 = 0$
j. $x^3 - \frac{14}{40}x^2 + \frac{1}{40}x = 0$ or $40x^3 - 14x^2 + x = 0$

72. a. (3)
b. (5)
c. (1)
d. (2)
e. (4)
f. (6)

Mixed Review

73. $3(n + 8)$
74. $3(n + 8)$
75. $\frac{n - 15}{2}$
76. $(a + b)^2$
77. $a^2 + b^2$
78. $a^2 - b^2$
79. $2(2y - 1)(y + 3)$
80. $(2x - y)(2x + y)$
81. $(2y - 3)(3y + 2)$

:TICE/ASSESS

UIZ

ıto an equation and solve.
are of a certain number is
ter than the number. Find
ıber.
the number.
+ 12
− 12 = 0
$x - 4) = 0$
or $x = 4$
duct of two consecutive,
even integers is 24. Find the
.
the smaller number. The
umber is $x + 2$.
$) = 24$
$= 24$
$- 24 = 0$
$x + 6) = 0$
or $x = -6$
itive solution $x = 4$ is the one
d. Hence the two numbers are

ıent Guide
1–28
ısion 29–34

ed Review to maintain skills.

Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

Practice Multiple Choice

Choose the best answer.

1. Solve.

$6x^2 = 29x - 35$

A $x = -1$
B $x = 10\frac{2}{3}$
C $x = -\frac{7}{3}$ and $-\frac{5}{2}$
D $x = \frac{7}{3}$ and $\frac{5}{2}$

2. The product of two consecutive odd integers is 483. Which equation could be used to solve for the integers?

F $x(x + 1) = 483$
G $x(x + 3) = 483$
H $x(x + 2) = 483$
J None of the above.

6-9 Exercises

A

Translate to an equation and solve.

1. If you subtract a number from four times its square, the result is three.
2. Fifteen more than the square of a number is eight times the number.
3. Eight more than the square of a number is six times the number.
4. If seven is added to the square of a number, the result is 32.
5. The product of two consecutive integers is 182.
6. The product of two consecutive integers is 56.
7. The product of two consecutive even integers is 168.
8. The product of two consecutive even integers is 224.
9. The product of two consecutive odd integers is 255.
10. The product of two consecutive odd integers is 143.
11. The length of a rectangle is 4 m greater than the width. The area of the rectangle is 96 m^2. Find the length and width.
12. The length of a rectangle is 5 cm greater than the width. The area of the rectangle is 84 cm^2. Find the length and width.
13. The number of square feet in the area of a square is 5 more than the number of feet in the perimeter of the square. Find the length of a side.
14. The number of inches in the perimeter of a square is 3 more than the number of square inches in its area. Find the length of a side.
15. The base of a triangle is 10 cm greater than the height. The area is 28 cm^2. Find the height and base.
16. The height of a triangle is 8 m less than the base. The area is 10 m^2. Find the height and base.
17. If the sides of a square are lengthened by 3 m, the area becomes 81 m^2. Find the length of a side of the original square.
18. If the sides of a square are lengthened by 7 in., the area becomes 121 $in.^2$. Find the length of a side of the original square.
19. The sum of the squares of two consecutive odd positive integers is 74.
20. The sum of the squares of two consecutive odd positive integers is 130.

21. **TEST PREP** Which equation would be used to solve the following problem: The sum of the squares of two consecutive even positive integers is 52?

A. $(x + x + 2)^2 = 52$
B. $x^2 + (x + 1)^2 = 52$
C. $(x + x + 1)^2 = 52$
D. $x^2 + (x + 2)^2 = 52$

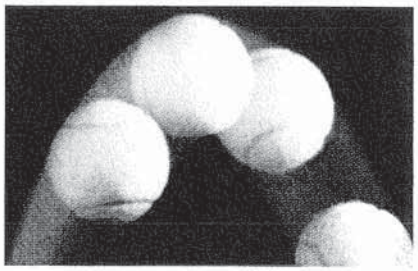
1. D; Algebra 11.0
2. H; Algebra 11.0

Exercises

1. $\frac{-3}{4}$ or 1
2. 3 or 5
3. 2 or 4
4. −5 or 5
5. 13 and 14, −13 and −14
6. 7 and 8, −7 and −8
7. 12 and 14, −12 and −14
8. 14 and 16, −14 and −16
9. 15 and 17, −15 and −17
10. 11 and 13, −11 and −13
11. 12 m, 8 m
12. 12 cm, 7 cm
13. 5 ft
14. 1 in. or 3 in.
15. 4 cm, 14 cm
16. 2 m, 10 m
17. 6 m
18. 4 in.
19. 5 and 7
20. 7 and 9
21. D

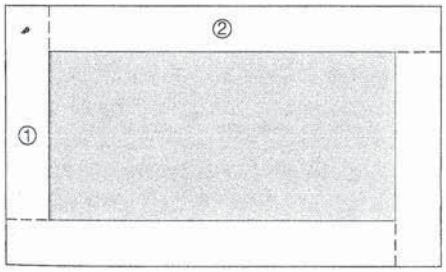
B

22. The sum of 7 times a positive number and 1 is the same as the square of 1 more than the number.
23. The sum of 6 times a positive number and 1 is the same as the square of 1 less than the number.
24. The cube of a number is the same as twice the square of the number.
25. ***Error Analysis*** A student read the following exercise: *The difference of a number squared and two is 38.* The student wrote the equation $(x - 2)^2 = 38$. What error did the student make?
26. Mark launched a model rocket with an initial speed of 180 ft per second. After how many seconds will Mark's rocket reach a height of 464 ft?
    a. The formula $h = rt - 16t^2$ gives the height of an object projected upward at a rate of r feet per second after t seconds. Rewrite this equation substituting the data you are given.
    b. Solve the equation and answer the problem.
    c. After how many seconds will it be at that height again?
27. When distance is measured in meters and the speed of the object is measured in meters per second, the formula in Exercise 26 becomes $h = rt - 4.9t^2$. A tennis ball is bounced upward with speed of 9.8 m per second.
    a. After how many seconds will the ball reach a height of 4.9 m?
    b. After how many seconds will the ball hit the ground?



Use the formula in Exercise 27 to find the height of the ball at 0.5 seconds and 1.5 seconds.

28. ***Multi-Step Problem*** A cement walk of constant width is built around a 20 ft × 40 ft rectangular pool. The total area of the pool and walk is 1500 ft^2. Find the width of the walk. Let w be the width of the walkway.
    a. Copy and complete the diagram below showing all of the data you are given and the unknown dimensions.



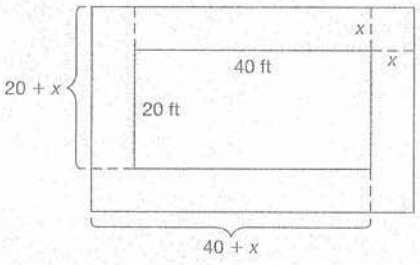
    b. What is the total area of the walkway?
    c. Write an expression for the area of section 1.
    d. Write an expression for the area of section 2.
    e. Write an expression that represents the total area of the walkway.
    f. Solve the equation and answer the problem.

Exercises

22. $7x + 1 = (x + 1)^2$, $x = 5$
23. $6x + 1 = (x - 1)^2$, $x = 8$
24. $x^3 = 2x^2$, $x = 0$ or $x = 2$
25. The student subtracted two from the number before squaring instead of squaring the number then subtracting two.
26. a. $464 = 180t - 16t^2$
    b. 4 s
    c. $7\frac{1}{4}$ s
27. a. 1 second
    b. 2 seconds

Photo caption: 3.675 m at 0.5 s and at 1.5 s

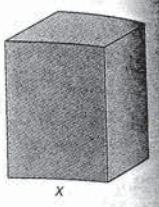
28. a.



    b. $1500 - (40 \cdot 20) = 700$ ft^2
    c. $x(x + 20)$
    d. $x(x + 40)$
    e. $2[x(x + 20) + x(x + 40)]$
    f. The width of the walkway is 5 ft.

29. ***Write a Convincing Argument*** Solve the problem below. Then write an argument that would convince a classmate that your solution is correct.

The total surface area of a box is 350 in.^{2} The box is 9 in. high and has a square base. Find the length of the side of the base.



Challenge

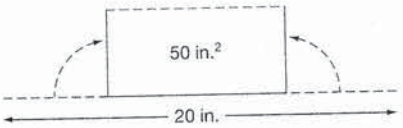
30. Find two consecutive positive numbers such that the product of the sum and difference of the numbers plus eight is the sum of their squares.

31. Find two consecutive positive numbers such that the sum of the squares of the numbers is one plus the product of the numbers, plus the quantity eight times the larger number.

32. The pages of a book are 15 cm by 20 cm. Margins of equal width surround the printing on each page and comprise one half the area of the page. Find the width of the margins.

33. A rectangular piece of cardboard is twice as long as it is wide. A 4-cm square is cut out of each corner, and the sides are turned up to make a box. The volume of the box is 616 cm^3. Find the original dimensions of the cardboard.

34. An open rectangular gutter is made by turning up the sides of a piece of metal 20 in. wide. The area of the cross section of the gutter is 50 in.^{2} Find the depth of the gutter.



Mixed Review

Write using scientific notation. **35.** 117, 203 **36.** 0.00559 *5-4*

Simplify. **37.** $(10m^2)^4$ **38.** $(5x^5)(x^2)(16x)$ **39.** $(3a^3b^2)(-5ab^3)$ *5-2, 5-3*

Solve. **40.** What number is 80% of 30?

41. 34 is 5% of what number? *3-10*

Factor. **42.** $a^2 - 8a + 16$ **43.** $6c^2 - 15c - 9$

44. $x^4 - x^2$ **45.** $2m^3 - 4m^2 - m + 2$

46. $36y^4 - 63$ **47.** $5x^2 + 70x + 245$ *6-2, 6-3, 6-5, 6-7*

Exercises

29. 7 in.
30. 2, 3
31. 8, 9
32. $2\frac{1}{2}$ cm
33. 30 cm by 15 cm
34. 5 in.

Mixed Review

35. 1.17203×10^5
36. 5.59×10^{-3}
37. $10{,}000m^8$
38. $80x^8$
39. $-15a^4b^5$
40. 24
41. 680
42. $(a - 4)^2$
43. $3(2c + 1)(c - 3)$
44. $x^2(x - 1)(x + 1)$
45. $(2m^2 - 1)(m - 2)$
46. $9(4y^4 - 7)$
47. $5(x + 7)^2$

6 Chapter Wrap Up

6-1

Factoring is the reverse of multiplying. Some of the possible **factorizations** for $12a^2b$ are $(12a)(ab)$, $(6a^2)(2b)$, $(3a)(4ab)$, and $12(a^2b)$. To factor many polynomials, factor out the greatest common factor of the terms.

$$6a^2b - 12ab^2 = (6ab)a - (6ab)(2b) = 6ab(a - 2b)$$

Find three factorizations for each monomial.

1. $-10x^2$ **2.** $36x^5$

Factor.

3. $x^2 - 3x$ **4.** $6y^3 + 12y^2 + 3y$

5. $8x^6 - 32x^5 + 4x^4$ **6.** $6a^4b^4 - 2a^3b + 8a^2$

6-2

A binomial is a **difference of two squares** if both terms of the binomial are squares, and there is a minus sign between the two terms. The difference of two squares, $A^2 - B^2$, factors as two binomials $(A - B)(A + B)$.

Which of the following are differences of squares?

7. $4x^2 - 8y^2$ **8.** $-25 + 81a^2$

Factor.

9. $9x^2 - 4$ **10.** $4x^2 - 25$ **11.** $2x^2 - 50$

12. $3x^2 - 27$ **13.** $x^4 - 81$ **14.** $16x^4 - 1$

6-3

A **trinomial square** has three terms and is the square of a binomial.

$$A^2 + 2AB + B^2 = (A + B)^2$$
$$A^2 - 2AB + B^2 = (A - B)^2$$

Which of the following are trinomial squares?

15. $y^2 + 3y + 9$ **16.** $49a^2 - 112a + 16$

17. $c^2 + 12c + 36$ **18.** $4c^2 - 4c - 1$

Factor.

19. $x^2 - 6x + 9$ **20.** $x^2 + 14x + 49$ **21.** $9x^2 - 30x + 25$

22. $25x^2 - 20x + 4$ **23.** $18x^2 - 12x + 2$ **24.** $12x^2 + 60x + 75$

Key Terms

binomial expansion (p. 285)
difference of two squares (p. 266)
factoring by grouping (p. 281)
factoring completely (p. 268)
factoring polynomials (p. 283)
factorization (p. 262)
principle of zero products (p. 286)
relatively prime (p. 264)
root (p. 288)
trinomial squares (p. 270)

Chapter 6 Wrap Up

1. } Answers may vary.
2. }
3. $x(x - 3)$
4. $3y(2y^2 + 4y + 1)$
5. $4x^4(2x^2 - 8x + 1)$
6. $2a^2(3a^2b^4 - ab + 4)$
7. No
8. Yes
9. $(3x - 2)(3x + 2)$
10. $(2x - 5)(2x + 5)$
11. $2(x - 5)(x + 5)$
12. $3(x - 3)(x + 3)$
13. $(x^2 + 9)(x - 3)(x + 3)$
14. $(4x^2 + 1)(2x - 1)(2x + 1)$
15. No
16. No
17. Yes
18. No
19. $(x - 3)^2$
20. $(x + 7)^2$
21. $(3x - 5)^2$
22. $(5x - 2)^2$
23. $2(3x - 1)^2$
24. $3(2x + 5)^2$

Internet Activity On the Web

Look for extension problems for this chapter at the Prentice Hall Web site.
www.phschool.com

6-4

To factor a trinomial of the type $x^2 + bx + c$, think of FOIL in reverse. Look for factors of the constant term whose sum is the coefficient of the middle term.

Factor.

25. $x^2 - 8x + 15$ **26.** $x^2 + 4x - 12$ **27.** $y^2 + 9y + 20$

28. $b^2 - 3b - 18$ **29.** $m^2 + 15m + 56$ **30.** $p^2 - 7p - 8$

6-5

To factor a trinomial of the type $ax^2 + bx + c$, first check for common factors. Then test factors of the first and last terms to find the correct combination, using FOIL to test possible factorizations.

Factor.

31. $2x^2 - 7x - 4$ **32.** $6y^2 - 5y + 1$ **33.** $6a^2 - 28a - 48$

6-6

A polynomial with four terms can sometimes be **factored by grouping** and using the distributive property twice.

$$\begin{aligned} a^3 + 2a^2 + 3a + 6 &= (a^3 + 2a^2) + (3a + 6) \\ &= a^2(a + 2) + 3(a + 2) \\ &= (a + 2)(a^2 + 3) \end{aligned}$$

Factor by grouping.

34. $x^3 + x^2 + 3x + 3$ **35.** $x^4 + 4x^3 - 2x - 8$

36. $x^3 + 3x^2 - x - 3$ **37.** $6x^3 + 4x^2 + 3x + 2$

6-7

When you factor a polynomial, first look for a common factor. Then check the number of terms and look for special cases (difference of squares or trinomial squares). Always factor completely.

38. $7x^2 - 7$ **39.** $-75x^3 + 60x^2 - 12x$ **40.** $a^2 - 4a - 21$

41. $x^2 + 2x - 195$ **42.** $x^3 - 3x + 4x^2 - 12$ **43.** $1 - a^8$

6-8

To solve an equation using the **principle of zero products,** use the addition principle to get zero on one side of the equation and a factorization on the other side. Set each of the factors equal to 0 and solve separately. Check all solutions.

Solve.

44. $(x - 1)(x + 3) = 0$ **45.** $y(4y - 6) = 0$ **46.** $x^2 + 2x - 35 = 0$

47. $x^2 + x - 12 = 0$ **48.** $3x^2 - 2 = 5x$ **49.** $9x^2 = 16$

Chapter 6 Wrap Up

25. $(x - 5)(x - 3)$
26. $(x + 6)(x - 2)$
27. $(y + 4)(y + 5)$
28. $(b - 6)(b + 3)$
29. $(m + 7)(m + 8)$
30. $(p - 8)(p + 1)$
31. $(2x + 1)(x - 4)$
32. $(2y - 1)(3y - 1)$
33. $2(a - 6)(3a + 4)$
34. $(x^2 + 3)(x + 1)$
35. $(x^3 - 2)(x + 4)$
36. $(x - 1)(x + 1)(x + 3)$
37. $(3x + 2)(2x^2 + 1)$
38. $7(x - 1)(x + 1)$
39. $-3x(5x - 2)^2$
40. $(a - 7)(a + 3)$
41. $(x + 15)(x - 13)$
42. $(x + 4)(x^2 - 3)$
43. $(1 + a^4)(1 + a^2)(1 + a)(1 - a)$
44. $1, -3$
45. $0, \frac{3}{2}$
46. $-7, 5$
47. $-4, 3$
48. $-\frac{1}{3}, 2$
49. $\frac{4}{3}, -\frac{4}{3}$

6-9

You can use the Problem-Solving Guidelines to help you translate a problem into an equation and solve it. After you have solved the problem, check to see if your answer is reasonable.

Translate into an equation and find all solutions.

50. The square of a number is six more than the number. Find the number.

51. The product of two consecutive even integers is 288. Find the integers.

52. The product of two consecutive odd integers is 323. Find the integers.

53. Twice the square of a number is 10 more than the number. Find the number.

54. If the sides of a square picture frame are increased by 5 cm, the area becomes 289 cm^2. Find the length of a side of the original picture frame.

Test Item Analysis

Item	Lesson
1–3	6-1
4–6	6-2
7–9	6-3
10–12	6-4
13–15	6-5
16, 17	6-6
18–20	6-7
21–24	6-8
25–27	6-9

6 Chapter Assessment

Factor

1. $x^2 - 5x$

2. $6x^3 + 9x^2 - 3x$

3. $4y^4 - 8y^3 + 6y^2$

4. $4x^2 - 9$

5. $3x^2 - 75$

6. $3x^4 - 48$

7. $x^2 - 10x + 25$

8. $49x^2 - 84x + 36$

9. $45x^2 + 60x + 20$

10. $x^2 - 7x + 10$

11. $x^2 - x - 12$

12. $x^3 + 2x^2 - 3x$

13. $4x^2 - 4x - 15$

14. $5x^2 - 26x + 5$

15. $10x^2 + 28x - 48$

16. $x^3 + x^2 + 2x + 2$

17. $x^4 + 2x^3 - 3x - 6$

18. $6x^3 + 9x^2 - 15x$

19. $80 - 5x^4$

20. $y^5 - 8y^4 + 15y^3$

Solve.

21. $x^2 - x - 20 = 0$

22. $2x^2 + 7x = 15$

23. $4a^2 = 25$

24. $x(x - 3) = 28$

Translate to an equation and find all solutions.

25. Find the number whose square is 24 more than five times the number.

26. The length of a rectangle is 6 m more than the width. The area of the rectangle is 40 m^2. Find the length and the width.

27. The product of two consecutive even integers is 528. Find the integers.

Chapter 6 Wrap Up

50. 3 or −2
51. −18, −16 or 16, 18
52. −19, −17 or 17, 19
53. $\frac{5}{2}$ or −2
54. 12 cm

Chapter 6 Assessment

1. $x(x - 5)$
2. $3x(2x^2 + 3x - 1)$
3. $2y^2(2y^2 - 4y + 3)$
4. $(2x + 3)(2x - 3)$
5. $3(x + 5)(x - 5)$
6. $3(x^2 + 4)(x + 2)(x - 2)$
7. $(x - 5)^2$
8. $(7x - 6)^2$
9. $5(3x + 2)^2$
10. $(x - 2)(x - 5)$
11. $(x + 3)(x - 4)$
12. $x(x - 1)(x + 3)$
13. $(2x - 5)(2x + 3)$
14. $(x - 5)(5x - 1)$
15. $2(x + 4)(5x - 6)$
16. $(x^2 + 2)(x + 1)$
17. $(x^3 - 3)(x + 2)$
18. $3x(x - 1)(2x + 5)$
19. $5(4 + x^2)(2 + x)(2 - x)$
20. $y^3(y - 3)(y - 5)$
21. −4, 5
22. $\frac{3}{2}$, −5
23. $\frac{5}{2}$, $-\frac{5}{2}$
24. 7, −4
25. 8, −3
26. 10 m, 4 m
27. −24, −22, or 22, 24