

Union of Sets

What if an element is a member of both sets, then it needs to be listed only once in the union of those two sets.

Point
Venn diagrams are called Venn diagrams after the logician John Venn (1834–1923).

Worked Examples

Let $A = \{0, 1, 2, 3\}$ and $B = \{3, 5\}$.
 $A \cup B = \{0, 1, 2, 3, 5\}$
 Let $R = \{1, 3, 5\}$. Find $R \cup R$.
 $R \cup R = \{1, 3, 5\} = R$
 Let T be the set of positive divisors of 4.
 $T = \{1, 2, 4\}$.
 $T \cup T = \{1, 2, 4\}$

Enrichment

Students check to see whether the commutative and associative properties hold for intersection and union of sets.

Commutative properties
 $A \cap B = B \cap A$
 $A \cup B = B \cup A$

Associative properties
 $(A \cap B) \cap C = A \cap (B \cap C)$
 $(A \cup B) \cup C = A \cup (B \cup C)$

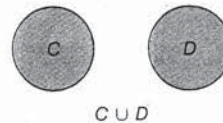
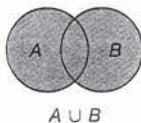
PART 3 Union of Sets

Objective: Find unions of sets.

The **union** of two sets A and B , written $A \cup B$, is the set of all members that are in A or in B (or in both). In other words, we form the union of two sets by putting them together. In the diagram below, the circles represent the sets. The shaded region represents the union of the sets.

Reading Math

Read $A \cup B$ as *A union B*.



EXAMPLE 6 Let $A = \{2, 3, 4\}$ and $B = \{3, 5, 7\}$. Find $A \cup B$.

The members 2, 3, 4, 5, and 7 are either in A or in B (or in both).

$$A \cup B = \{2, 3, 4, 5, 7\}$$

Try This Let $W = \{-3, -2, 0, 4\}$, $X = \{-4, -2, 5, 6\}$, and $Y = \{1, 3, 5\}$.

m. Find $W \cup X$. n. Find $W \cup Y$.

We can find unions involving infinite sets or the empty set.

EXAMPLE 7 Let $P = \{-2, -1, 4\}$. Find $P \cup \emptyset$.

The members -2 , -1 , and 4 are in P or in \emptyset .

$$P \cup \emptyset = \{-2, -1, 4\} = P$$

EXAMPLE 8 Let $S = \{4, 6, 8, 10\}$ and E be the set of even numbers.

Find $S \cup E$.

All even numbers are in S or in E .

$$S \cup E = E$$

Try This Let $M = \{1, 3, 5\}$, E be the set of even whole numbers, and D be the set of odd whole numbers.

o. Find $M \cup D$. p. Find $E \cup D$. q. Find $M \cup \emptyset$.

Try This

- m. $\{-4, -3, -2, 0, 4, 5, 6\}$
 n. $\{-3, -2, 0, 1, 3, 4, 5\}$
 o. D
 p. $\{x | x \text{ is a whole number}\}$
 q. M

Exercises

- (a) $A = \{0, 1, 2, 3\}$
 (b) $A = \{x | x \text{ is a whole number and } x < 4\}$
- (a) $F = \{12, 13, 14, \dots\}$
 (b) $F = \{x | x \text{ is an integer and } x \geq 12\}$
- (a) $N = \{-1, -2, -3, -4\}$
 (b) $N = \{x | x \text{ is an integer and } -5 < x < 0\}$
- (a) $P = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
 (b) $P = \{x | x \text{ is a prime number and } x < 30\}$
- (a) $H = \{3, 6, 9, 12, 15, 18, 21\}$
 (b) $H = \{x | x \text{ is a multiple of 3 and } 0 < x \leq 21\}$
- (a) $R = \{4, 8, 12, 16, \dots\}$
 (b) $R = \{x | x \text{ is a multiple of 4 and } x > 0\}$
- (a) $E = \{2, 4, 6, 8, \dots\}$
 (b) $E = \{x | x \text{ is an even integer and } x > 0\}$

9-1 Exercises



Extra Help
On the Web

Look for worked-out examples at the Prentice Hall Web site.

www.phschool.com

A

Write using (a) roster notation and (b) set-builder notation.

- the set A of whole numbers less than 4
- the set F of integers greater than or equal to 12
- the set N of negative integers greater than -5
- the set P of prime numbers less than 30
- the set H of positive multiples of 3 less than or equal to 21
- the set R of positive multiples of 4
- the set E of positive even integers
- the set T of positive factors of 12
- the set M of even prime numbers
- the set S of integers that are perfect squares less than 20

Let $A = \{-4, -3, -2, -1, 0\}$, $B = \{0, 1, 2\}$, $C = \{1, 2, 3, 4, 5\}$, and $D = \{4, 5, 6, 7, 8, 9, 10\}$. Find each of the following.

- $B \cap C$
- $A \cap B$
- $C \cap D$
- $C \cap B$
- $A \cap C$
- $B \cap D$
- $A \cap \emptyset$
- $D \cap \emptyset$

19. Let W be the set of whole numbers and E the set of even numbers. Find $W \cap E$.

20. Let Z be the set of integers and D the set of odd positive integers. Find $D \cap Z$.

Let $P = \{-5, -4, -3, -2, -1, 0\}$, $Q = \{-2, -1, 0\}$, $R = \{-1, 0, 1, 2, 3, 4\}$, and $S = \{5, 6, 7, 8\}$. Find each of the following.

- $P \cup Q$
- $Q \cup R$
- $S \cup P$
- $Q \cup S$
- $R \cup S$
- $R \cup P$
- $P \cup \emptyset$
- $S \cup \emptyset$

29. Let W be the set of whole numbers and E the set of even positive numbers. Find $W \cup E$.

30. Let Z be the set of integers and D the set of odd positive integers. Find $D \cup Z$.

B

Let E be the set of even numbers, J the set of integers less than -9 , and P the set of odd numbers between 7 and 29. To say that -9 is an element of Z , we write $-9 \in Z$. To say that $\frac{5}{8}$ is not a member of Z , we write $\frac{5}{8} \notin Z$.

Tell whether each of the following is true or false.

- $2 \in E$
- $-7 \in J$
- $19 \in P$
- $0 \in J$
- $-10 \in E$
- $23 \in P$
- $5 \notin P$
- $16 \notin E$
- $-8 \in P$
- $10 \notin P$
- $-5 \notin J$
- $0 \notin E$

3. PRACTICE/ASSESS

LESSON QUIZ

Let $A = \{1, 2, 3, 4\}$, $B = \{0, 2, 4, 6\}$, $C = \{1, 3, 5, 7\}$, Z be the integers.

- Find $A \cap B$.
{2, 4}
- Find $A \cap C$.
{1, 3}
- Find $B \cap C$.
 \emptyset
- Find $A \cup B$.
{0, 1, 2, 3, 4, 6}
- Find $B \cup C$.
{0, 1, 2, 3, 4, 5, 6, 7}
- Find $A \cap \{x | x \in Z \text{ and } x < 3\}$.
{1, 2}
- Find $A \cup \emptyset$.
 A

Assignment Guide

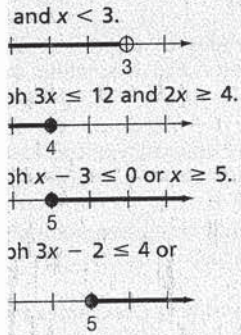
To provide flexible scheduling, a lesson can be split into parts.

- ▼ Core 1–10
Extension 31–42
- ▼ Core 11–20
Extension 43, 46, 47, 51
- ▼ Core 21–30
Extension 44, 45, 48–50, 5.

Use Mixed Review to maintain

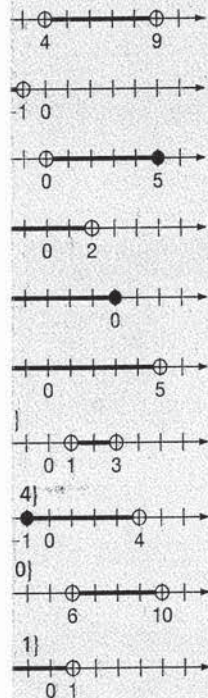
- (a) $T = \{1, 2, 3, 4, 6, 12\}$
- (b) $T = \{x | x \text{ is a factor of } 12 \text{ and } x > 0\}$
- (a) $m = \{2\}$
- (b) $m = \{x | x \text{ is an even prime number}\}$
- (a) $s = \{0, 1, 4, 9, 16\}$
- (b) $s = \{x | x \text{ is an integer, a perfect square, and } x < 20\}$
- {1, 2}
- {0}
- {4, 5}
- {1, 2}
- \emptyset
- \emptyset
- {1, 2, 3, 4, 6, 8, ...} or E
- {1, 3, 5, 7, 9, ...} or D
- $\{-5, -4, -3, -2, -1, 0\}$ or P
- $\{-2, -1, 0, 1, 2, 3, 4\}$
- $\{-5, -4, -3, -2, -1, 0, 5, 6, 7, 8\}$
- $\{-2, -1, 0, 5, 6, 7, 8\}$
- $\{-1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$
- $\{-5, -4, -3, -2, -1, 0\}$ or P
- $\{5, 6, 7, 8\}$ or S
- $\{0, 1, 2, 3, \dots\}$ or W
- Z
- T
- F
- T
- F
- T
- T
- F
- F
- T
- T
- F

ASSESS



Guide
flexible scheduling, this
split into parts.
34-36, 38, 41, 44-46
8, 49
, 37, 39, 40, 42, 43, 47
0-54

view to maintain skills.



Extra Help On the Web

Look for worked-out
examples at the Prentice
Hall Web site.
www.phschool.com



Practice Multiple Choice

Choose the best
answer.

1. Let $R = \{-1, 1, 2, 5, 7\}$ and let $S = \{-7, 1, 4, 7\}$. Find $R \cap S$.

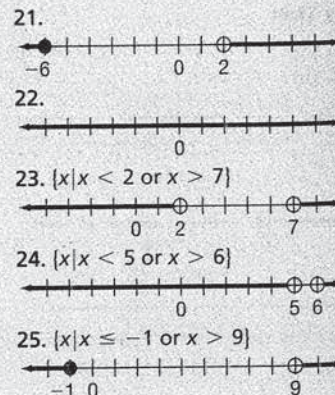
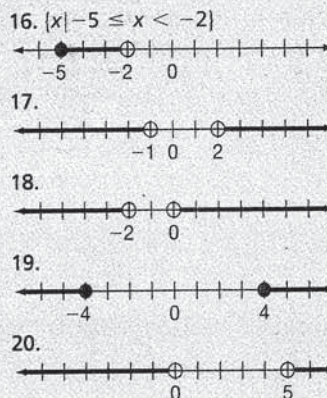
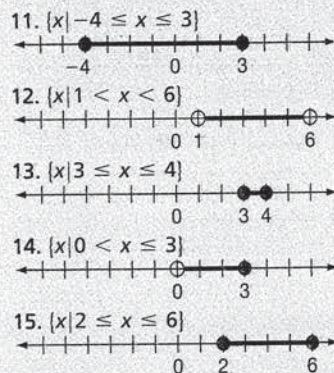
- A $\{-1, -7\}$
B $\{1, 2, 4, 5, 7\}$
C $\{-7, -1, 1, 2, 4, 5, 7\}$
D $\{1, 7\}$

2. If both parts of a
two-part conjunction
statement are true,
which of the
following statements
is true?

- The conjunction
F is never true.
G is sometimes true.
H is always true.
J We cannot
determine the truth
of the conjunction.

1. D; Algebra 17.0
2. H; Algebra 25.3

408 Chapter 9 Inequalities and Absolute Value



9-2 Exercises

A
Mental Math Graph these conjunctions.

1. $4 < x$ and $x < 9$ 2. $-3 < x$ and $x < -1$
3. $0 < x$ and $x \leq 5$ 4. $-3 \leq x$ and $x < 2$
5. $-7 \leq x$ and $x \leq 0$ 6. $-4 \leq x < 5$

Solve and graph.

7. $3 < x + 2 < 5$ 8. $-4 \leq x - 3 < 1$
9. $12 < 2x < 20$ 10. $-18 < 3x < 3$
11. $-20 \leq 5x \leq 15$ 12. $7 < 2x + 5 < 17$
13. $19 \leq 5x + 4 \leq 24$ 14. $-5 < 7x - 5 \leq 16$
15. $-11 \leq -2x + 1 \leq -3$ 16. $-19 \leq 5x + 6 < -4$

Mental Math Graph these disjunctions.

17. $x < -1$ or $x > 2$ 18. $x < -2$ or $x > 0$
19. $x \leq -4$ or $x \geq 4$ 20. $x < 0$ or $x > 5$
21. $x \leq -6$ or $x > 2$ 22. $x < 1$ or $x > -3$

Solve and graph.

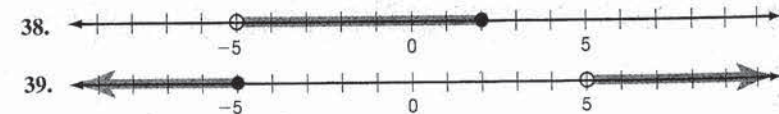
23. $x + 8 < 10$ or $x - 5 > 2$ 24. $x - 2 < 3$ or $x + 3 > 9$
25. $x + 1 \leq 0$ or $x - 6 > 3$ 26. $5x < -20$ or $3x > 12$
27. $4x < 16$ or $3x > 15$ 28. $-2x > -14$ or $3x > 21$
29. $2x + 7 \leq -3$ or $5x - 9 > 6$ 30. $4x - 9 \leq -17$ or $2x + 6 > 8$
31. **TEST PREP** The solution of which inequality is $x < -10$ or $x > 4$?
A. $5x > 20$ or $2x < -20$ B. $-15 < x + 5 < 9$
C. $x + 3 > 7$ and $x + 10 < 0$ D. $-3x < -30$ or $x - 7 > -3$

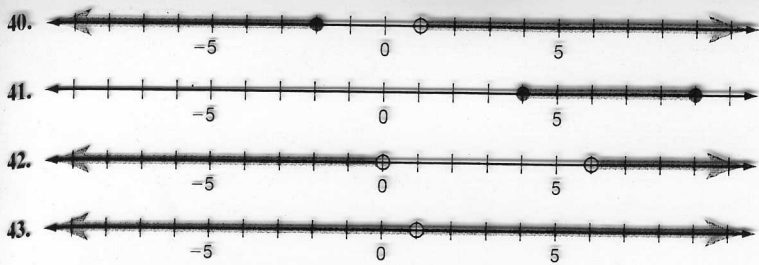
B

Solve.

32. $x + 2 < 7$ or
 $x - 1 > -4$ 33. $x - 5 < -3$ or
 $x + 8 > 7$
34. $3x + 12 \leq 6$ and
 $2x - 4 > 2$ 35. $-2x + 1 > 5$ and
 $-2x + 10 > 2$
36. $-6 > -3x - 6$ and
 $-3x - 6 \geq 6$ 37. $3x - 8 < 13$ or
 $-3x + 10 > 5$

Write the inequality shown by each graph.



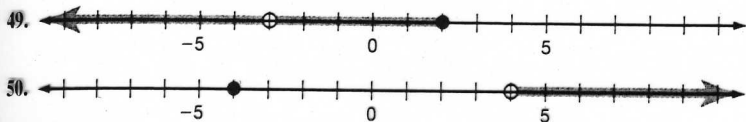


Write a compound sentence for each situation.

44. The length of an adult triceratops dinosaur was between 20 feet and 25 feet. (Let l be the length in feet.)
 45. The Mesozoic era was 145 million years ago plus or minus 85 million years.
 46. Gold must be at least 1063°C and not more than 2808°C to be in liquid form.
 47. Mercury is not a liquid if its temperature is below -39°C or above 357°C .
 48. **Critical Thinking** If $a > b$ and $c > d$, is $ac > bd$ for all rational values of a, b, c , and d ? Explain.

Challenge

Write a conjunction or disjunction to describe each graph.



Solve and graph.

51. $(x - 3)(x + 5) \geq 0$ 52. $(x + 8)(x - 4) < 0$
 53. $x^2 + x - 2 > 0$ 54. $x^2 - x - 6 \leq 0$



Gold is liquid between 1063°C and 2808°C . Write an inequality for temperatures where gold is not liquid.

Mixed Review

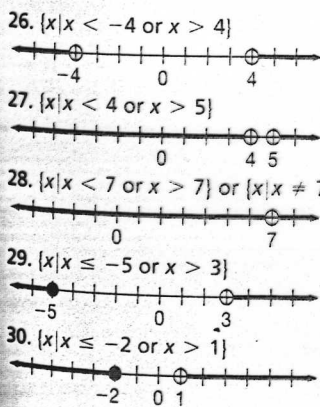
Identify the degree of each term and the degree of the polynomial.

55. $9a^2 - 25$ 56. $7x^3y^2 + 21x^2y - 14x + 5$ 57. $2a^2b^3c^4 = b^6$ 5-5

Write an equation in slope-intercept form for the line that contains the given pair of points. 58. $(-3, 1), (4, 2)$ 59. $(0, 5), (-2, 1)$ 7-6

- Solve. 60. $x + y = -1$ 61. $4x - 2y = 18$ 62. $4x + 5y = -1$
 $x - y = 3$ $x + 3y = -20$ $2x - 3y = 5$ 8-1, 8-2, 8-3

63. A collection of nickels and dimes is worth \$3.40. There are 41 coins in all. How many are nickels and how many are dimes? 8-6
 64. The difference of two numbers is 8; the product is 65. Find the numbers. 8-6



31. A
 32. all numbers on the number line
 33. all numbers on the number line
 34. \emptyset
 35. $\{x|x < -2\}$
 36. \emptyset
 37. $\{x|x < 7\}$
 38. $-5 < x$ and $x \leq 2$ or $-5 < x \leq 2$
 39. $x \leq -5$ or $x > 5$
 40. $x \leq -2$ or $x > 1$
 41. $4 \leq x$ and $x \leq 9$ or $4 \leq x \leq 9$
 42. $x < 0$ or $x > 6$

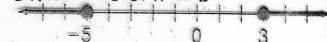
43. $x < 1$ or $x > 1$
 44. $20 < l$ or $l < 25$ or $20 < l < 25$
 45. $-230,000,000 \leq M$ and $M \leq -60,000,000$ or $-230,000,000 \leq M \leq -60,000,000$
 46. $1063 \leq G$ and $G \leq 2808$ or $1063 \leq G \leq 2808$
 Photo caption: $G \leq 1063$ or $G \geq 2808$
 47. $L < -39$ or $L > 357$

48. No, suppose $a = -1, b = -2,$ and $d = 0$. Then $ac = -1$ and -1 is not greater than 0.

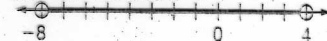
49. $x \leq 2$ and $x \neq -3$ OR $x < -3$ or $(-3 < x$ and $x \leq 2)$

50. $x = -4$ or $x > 4$

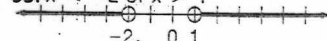
51. $x \leq -5$ or $x \geq 3$



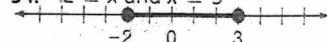
52. $-8 < x$ and $x < 4$



53. $x < -2$ or $x > 1$



54. $-2 \leq x$ and $x \leq 3$



Mixed Review

55. 2, 0; 2
 56. 5, 3, 1, 0; 5
 57. 9, 6; 9
 58. $y = \frac{1}{7}x + \frac{10}{7}$
 59. $y = 2x + 5$
 60. $(1, -2)$
 61. $(1, -7)$
 62. $(1, -1)$
 63. 14 nickels; 27 dimes
 64. 5; 13 or -5, -13

CTICE/ASSESS

uiz

$$x - 7 = 2$$

or $x = 5$

$$4x - 10 = 2$$

or $x = 2$

ment Guide

1-33
nsion 34-37

ked Review to maintain skills.



Extra Help On the Web

Look for worked-out examples at the Prentice Hall Web site.

www.phschool.com

Quick Review

To review solving equations, see Lessons 3-3 and 3-5.



Self-Test On the Web

Check your progress. Look for a self-test at the Prentice Hall Web site. www.phschool.com

Exercises

- $\{9, -27\}$
- $\{13, -5\}$
- $\{-17, -5\}$
- $\{-16, 30\}$
- \emptyset
- $\{-15, -19\}$
- $\{5, -1\}$
- $\{4, 3/2\}$

9-3 Exercises

A

Solve.

1. $|x + 9| = 18$
2. $|x - 4| = 9$
3. $|x + 11| = 6$
4. $|m - 7| = 23$
5. $|x - 10| = -8$
6. $|x + 17| = 2$
7. $|2x - 4| = 6$
8. $|4b - 11| = 5$
9. $|7x - 2| = 5$
10. $|8x + 3| = -27$
11. $|5x - 9| = 1$
12. $|4x + 3| = 67$
13. $|2y - 6| = -9$
14. $|4x + 3| = -5$
15. $|3x + 1| = 0.5$
16. $|5y + 8| = \frac{1}{2}$
17. $|2r - 1| = \frac{1}{4}$
18. $|\frac{1}{3}x - 9| = 10$
19. $|0.2x + 1| = 0.8$
20. $|4.2x - 1.4| = 7$
21. $|5x + 0.2| = 1.2$

B

Solve.

22. $|2x + 5| - 9 = 12$
 23. $|3y - 2| + 4 = 21$
 24. $|2 - a| - 3 = 1$
 25. $8 - |1 - y| = 7$
 26. $3|y + 6| = 6$
 27. $4|t + 3| = 16$
 28. $8 + |2c - 1| = 4$
 29. $3|b - 2| + 7 = 10$
 30. $4|3 - z| - 8 = 8$
 31. $10 + |5x + 2| = 7$
 32. $6 - |3y - 2| = 10$
 33. $-|x + 1| = -2$
34. **Critical Thinking** Make one change in the equation $|x + 3| = 5$ so that the solution set of the new equation is $\{0, -6\}$.

Challenge

Solve.

35. $|x - 4| = x - 4$
36. $|3x| = -|4x - 1|$
37. $|2y| = |3y + 2| + 1$

Mixed Review

Find the slope and y-intercept of each line.

38. $x - 2y = 1$
39. $-4x - y = 7$
40. $3x = y - 4$
41. $5y + 4 = 2x - 1$
42. $3x - 5 = -4y + 7$
- Factor. 43. $2x^3 - 4x^2 - 6x$
44. $2y^2 - 9y + 4$
45. $9a^2 - 4$
- Solve. 46. $3y + 4x = -5$
47. $2x + y = -2$
48. $3x - y = 2$
49. $(m + 2)(m - 1) = 0$
50. $a(2a + 6)(a - 5) = 0$
51. Ray is 42 years old. Eight years ago, Ray was twice as old as Roy. How old is Roy today?

- | | | | |
|--|----------------------------|---|-------------------------------|
| 9. $\{1, -\frac{3}{7}\}$ | 18. $\{57, -3\}$ | 30. $\{-1, 7\}$ | 40. $m = 3, b = 4$ |
| 10. \emptyset | 19. $\{-1, -9\}$ | 31. \emptyset | 41. $m = \frac{2}{5}, b = -1$ |
| 11. $\{2, \frac{8}{5}\}$ | 20. $\{2, -\frac{4}{3}\}$ | 32. \emptyset | 42. $m = -\frac{3}{4}, b = 3$ |
| 12. $\{16, -\frac{35}{2}\}$ | 21. $\{0.2, -0.28\}$ | 33. $\{1, -3\}$ | 43. $2x(x - 3)(x + 1)$ |
| 13. \emptyset | 22. $\{8, -13\}$ | 34. $ x + 3 = 3$ | 44. $(2y - 1)(y - 4)$ |
| 14. \emptyset | 23. $\{\frac{19}{3}, -5\}$ | 35. $\{x x \geq 4\}$ | 45. $(3a + 2)(3a - 2)$ |
| 15. $\{-\frac{1}{6}, -\frac{1}{2}\}$ | 24. $\{6, -2\}$ | 36. $\{1, \frac{1}{3}\}$ | 46. $(1, -3)$ |
| 16. $\{-\frac{3}{2}, -\frac{17}{10}\}$ | 25. $\{0, 2\}$ | 37. $\{-\frac{3}{5}, -1\}$ | 47. $(1, -4)$ |
| 17. $\{\frac{3}{8}, \frac{5}{8}\}$ | 26. $\{-4, -8\}$ | Mixed Review | 48. $(1, 1)$ |
| | 27. $\{1, -7\}$ | 38. $m = \frac{1}{2}, b = -\frac{1}{2}$ | 49. $1, -2$ |
| | 28. \emptyset | 39. $m = -4, b = -7$ | 50. $-3, 0, 5$ |
| | 29. $\{3, 1\}$ | | 51. 25 yr |

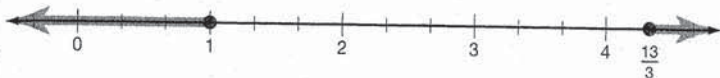
Solving Inequalities with Absolute Values $|A| > b$

To solve an inequality of the form $|A| > b$ where b is a positive number, we solve the disjunction $A < -b$ or $A > b$. A similar rule holds for $|A| \geq b$.

EXAMPLE 4 Solve and graph $|3x - 8| \geq 5$.

Solve the disjunction $3x - 8 \leq -5$ or $3x - 8 \geq 5$.

$$\begin{aligned} 3x - 8 &\leq -5 & \text{or} & & 3x - 8 &\geq 5 \\ 3x &\leq 3 & & & 3x &\geq 13 \\ x &\leq 1 & & & x &\geq \frac{13}{3} \end{aligned}$$



The solution set is $\{x | x \leq 1 \text{ or } x \geq \frac{13}{3}\}$.

Try This Solve and graph.

- h. $|2x| > 10$ i. $|x - 4| \geq 5$ j. $|2x + 4| \geq 16$

9-4 Exercises

A
Solve and graph.

- | | | | |
|------------------------|-----------------------|------------------------|----------------------|
| 1. $ x < 1$ | 2. $ t \leq 4.5$ | 3. $ 5x \leq 20$ | 4. $ 6x \leq 24$ |
| 5. $ 2x < 11$ | 6. $ 5y \leq 5$ | 7. $ 4t < 28$ | 8. $ 6x \leq 36$ |
| 9. $ 7x \leq 35$ | 10. $ x - 3 < 12$ | 11. $ x + 2 \leq 5$ | 12. $ x - 5 \leq 7$ |
| 13. $ x + 6 < 2$ | 14. $ 2y - 4 < 7$ | 15. $ 4y - 2 < 7$ | |
| 16. $ 3x + 4 \leq 10$ | 17. $ 2x + 1 \leq 5$ | 18. $ 4z + 3 \leq 15$ | |

In Example 2, suppose the diameter of the gear satisfies the following equation. What are the acceptable diameters for the gear?

19. $|d - 1| < 0.011$ 20. $|2d - 2| \leq 1.1$ 21. $|2d - 4| \leq 0.002$

Graph on a number line.

22. $|y| > 3$ 23. $|t| > 4$ 24. $|y| \geq 7$ 25. $|x| \geq 9$

26. **TEST PREP** Which inequality has the solution $-6 < t < 4$?

- A. $|t + 1| < 5$ B. $|t + 1| > 5$
C. $|t - 1| < 5$ D. $|t - 1| > 5$



**Extra Help
On the Web**

Look for worked-out examples at the Prentice Hall Web site.

www.phschool.com

3. PRACTICE/ASSES

Lesson Quiz

1. Graph $|x| \leq 3$ on the number line.



2. Solve and graph $|x - 1| < 2$.



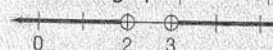
$$\{x | -2 < x < 4\}$$

3. Solve and graph $|2x| > 4$.



$$\{x | x < -2 \text{ or } x > 2\}$$

4. Solve and graph $|4x - 10| \leq 2$.



$$\{x | x < 2 \text{ or } x > 3\}$$

Assignment Guide

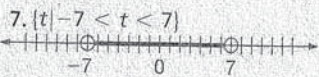
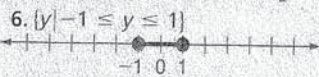
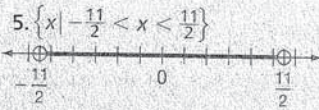
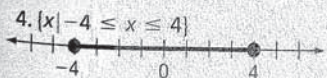
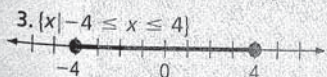
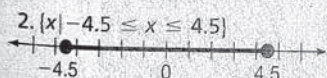
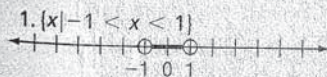
To provide flexible scheduling, this lesson can be split into parts:

▼ Core 1–21, 26, 40, 41, 45–47, Extension 43

▼ Core 22–25, 27–39, 42–47, Extension 44, 48–50

Use Mixed Review to maintain skills.

Exercises



8–19. See Additional Answers for graphs. Pgs 658–663

8. $|x| - 6 \leq x \leq 6$
9. $|x| - 5 \leq x \leq 5$

10. $|x| - 9 < x < 15$

11. $|x| - 7 \leq x \leq 3$

12. $|x| - 2 \leq x \leq 12$

13. $|x| - 8 < x < -4$

14. $\{y | -\frac{3}{2} < y < \frac{11}{2}\}$

15. $\{y | -\frac{5}{4} < y < \frac{9}{4}\}$

16. $\{x | -\frac{14}{3} \leq x \leq 2\}$

17. $|x| - 3 \leq x \leq 2$

18. $\{z | -\frac{9}{2} \leq z \leq 3\}$

19. $|d| 0.989 < d < 1.011$

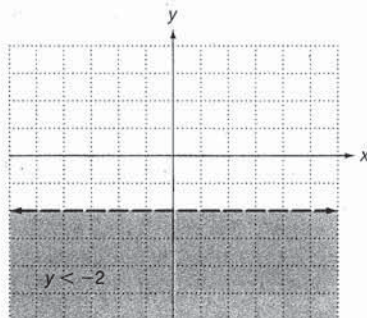
Answers continue on next page.

EXAMPLE 4 Graph $y < -2$ on a coordinate plane.

- (a) We graph the boundary line $y = -2$, using a dashed line to show that the points on the line are not solutions of $y < -2$.
 (b) We test a point that is not on the line, such as $(0, -3)$.

$$\begin{aligned} y &< -2 \\ -3 &< -2 \quad \text{Substituting } -3 \text{ for } y \end{aligned}$$

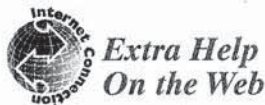
Since $-3 < -2$ is true, the half-plane containing $(0, -3)$ is the graph of the solution. We shade below the line to show that every point in that half-plane is a solution.



Try This Graph on a coordinate plane.

- c. $y > x - 1$ d. $2x + y \leq 4$
 e. $y - 3x < 0$ f. $x \geq 2$

9-5 Exercises



**Extra Help
On the Web**

Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A

- Determine whether $(-3, -5)$ is a solution of $-x - 3y < 18$.
- Determine whether $(5, -3)$ is a solution of $-2x + 4y \leq -2$.
- Determine whether $(\frac{1}{2}, -\frac{1}{4})$ is a solution of $7y - 9x > -3$.

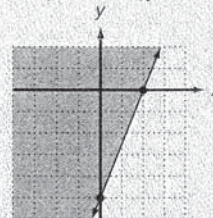
Graph on a coordinate plane.

- | | | |
|---------------------|-----------------------|-----------------------|
| 4. $x > 2y$ | 5. $x > 3y$ | 6. $y \leq x - 3$ |
| 7. $y \leq x - 5$ | 8. $y < x + 1$ | 9. $y < x + 4$ |
| 10. $y > 2$ | 11. $x \geq 3$ | 12. $x > 0$ |
| 13. $y \geq x - 2$ | 14. $y \geq x - 1$ | 15. $y \leq 2x - 1$ |
| 16. $y \leq 3x + 2$ | 17. $x + y \leq 3$ | 18. $x + y \leq 4$ |
| 19. $y \leq 0$ | 20. $y \geq -1$ | 21. $x \leq -2$ |
| 22. $x - y > 7$ | 23. $x - y > -2$ | 24. $x - 3y < 6$ |
| 25. $x - y < -10$ | 26. $2x + 3y \leq 12$ | 27. $5x + 4y \geq 20$ |
| 28. $y \geq 1 - 2x$ | 29. $y - 2x \leq -1$ | 30. $y + 4x > 0$ |
| 31. $y - x < 0$ | 32. $y > -3x$ | 33. $y < -5x$ |

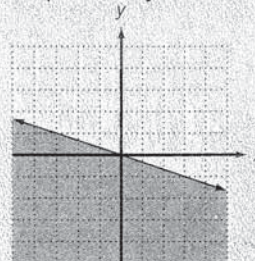
3. PRACTICE/ASSESS

LESSON QUIZ

- Determine whether $(2, 3)$ is a solution of the inequality $5x - 3y \leq 7$.
The point is a solution.
- Graph $5x - 2y \leq 10$ on the coordinate plane.



- Graph $2x + 6y \leq 0$ on the coordinate plane.



Assignment Guide

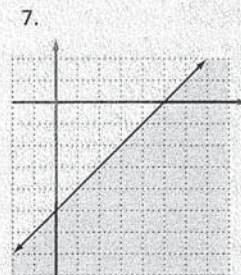
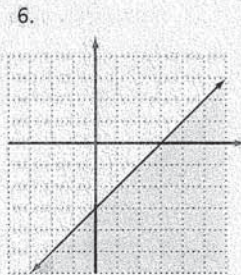
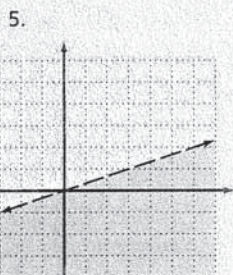
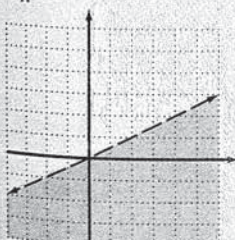
To provide flexible scheduling, this lesson can be split into parts.

- ▼ Core 1-3
Extension 34
- ▼ Core 4-33, 35-41
Extension 42-45

Use Mixed Review to maintain skills.

Exercises

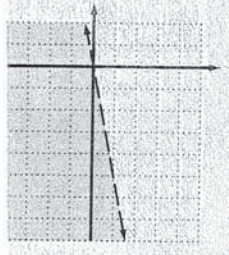
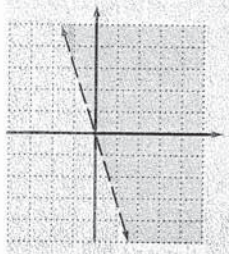
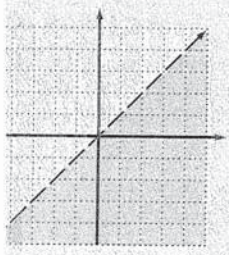
- No
- Yes
- No
-



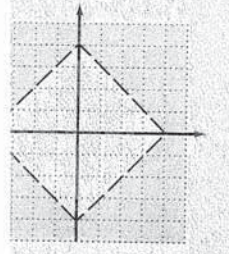
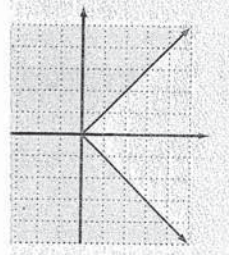
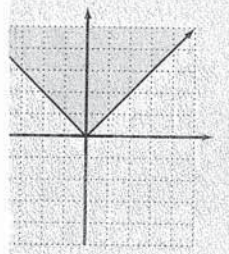
8-30. See Additional Answers.
 Answers continue on next page.

SEE PAGES 658-663

ises



$\leq x + 1$
 $\leq x - 4$
 $> x - 2$
 $\leq x + 4$
 $\leq x - 3$
 ≤ -2
 ≥ 0 and $y \leq 0$ or
 ≤ 0 and $y \geq 0$; $xy \leq 0$



Practice Multiple Choice

Choose the best answer.

1. Solve $|5x - 9| = 12$.

- A $x \in \{21, -3\}$
- B $x \in \{4\frac{1}{5}, -\frac{3}{5}\}$
- C $x \in \{6, 2\frac{2}{5}\}$
- D $x \in \{21, -3\}$

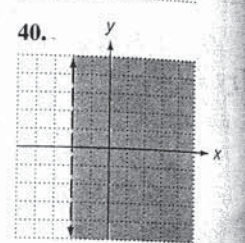
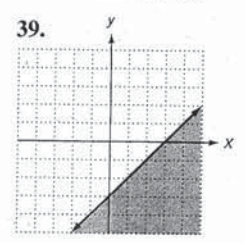
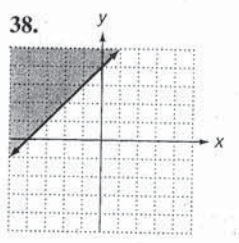
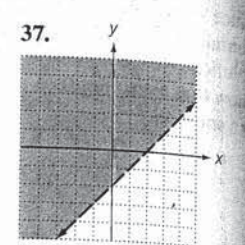
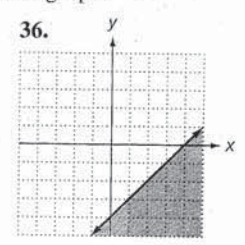
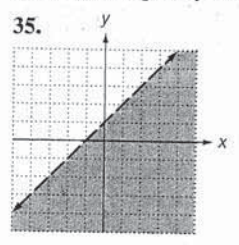
2. Solve $|2x - 1| \geq 3$

- F
- G
- H
- J

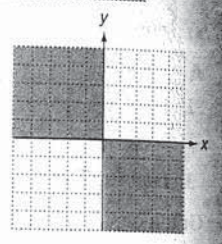
- 1. B; Algebra 3.0
- 2. J; Algebra 3.0

34. **TEST PREP** Which inequality has the same boundary line as $y \geq 2x - 3$?
- A. $2x + y \leq 3$
 - B. $y + 2x \geq 3$
 - C. $2x - y < -3$
 - D. $y - 2x > -3$
 - E. none of them

B
Write an inequality for each graph.



41. **Critical Thinking** What compound inequality is shown in the graph? Can you find another inequality represented by the graph?



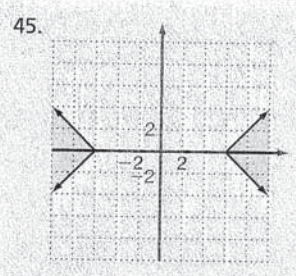
Challenge

Graph on a coordinate plane.

- 42. $|x| \leq y$
- 43. $|y| \geq x$
- 44. $|x| + |y| > 4$
- 45. $|x| - |y| \geq 6$

Mixed Review

- Graph on a number line.
- 46. $-5 \leq x$ and $x < 2$
 - 47. $-4 < x$ and $x < -1$
 - 48. $x \leq 0$ or $x > 3$ 4-1, 9-2
 - Solve. 49. $5 < x + 8 \leq 13$ 50. $2x + 1 \leq -7$ or $3x - 5 > 4$ 9-2
 - Factor. 51. $m^2 - 15m + 50$ 52. $2a^2 - 5a - 42$
 - 53. $2x^2 + 3xa + a^2$ 54. $6x^2 + 19x + 15$ 6-4, 6-5



Mixed Review

- 46.
- 47.
- 48.
- 49. $-3 < x \leq 5$
- 50. $x \leq -4$ or $x > 3$
- 51. $(m - 5)(m - 10)$
- 52. $(2a + 7)(a - 6)$

- 53. $(2x + a)(x + a)$
- 54. $(3x + 5)(2x + 3)$

9-6 Exercises



Look for worked-out examples at the Prentice Hall Web site.
www.phschool.com

A Solve each system by graphing.

- | | | |
|------------------|----------------------|-----------------------|
| 1. $x > 2 + y$ | 2. $y < 3x$ | 3. $y < x$ |
| $y > 1$ | $x + y < 4$ | $y < -x + 1$ |
| 4. $y > x$ | 5. $2y - y < 2$ | 6. $y > 4$ |
| $y > -x + 2$ | $x > 3$ | $2y + x > 5$ |
| 7. $y > 4x - 1$ | 8. $y > 5x + 2$ | 9. $x - 2y > 6$ |
| $y \leq -2x + 3$ | $y \leq -x + 1$ | $x + 2y \leq 4$ |
| 10. $2y - x > 5$ | 11. $2x - 3y \geq 9$ | 12. $3x - 2y \leq 8$ |
| $x + y \leq 4$ | $2y + x > 6$ | $2x + y > 6$ |
| 13. $x + y < 3$ | 14. $x - y < 3$ | 15. $5x + 2y \geq 12$ |
| $x - y \leq 4$ | $x + y \geq 4$ | $2x + 3y \leq 10$ |
| 16. $x \leq 4$ | 17. $y \leq x$ | 18. $y > 0$ |
| $x + y \leq 3$ | $x \geq -2$ | $x > y$ |
| $y \leq x$ | $y \leq 1 - x$ | $x + y \geq 2$ |
| 19. $y > 2x$ | 20. $y \leq x$ | 21. $x - y > 3$ |
| $y \leq x + 2$ | $y \geq -2$ | $x > 0$ |
| $x > -1$ | $y < 2x - 3$ | $x + y \leq 5$ |

22. **Multi-Step Problem** The school board is investigating ways to hire a faculty for the summer school program. They can hire teachers and aides. A minimum of 20 faculty members is needed to run the program, and there must be at least 12 teachers. For a proper teacher-to-aide ratio, the number of aides must be no more than twice the number of teachers. There can be no more than 50 faculty members altogether. (*Hint:* There cannot be a negative number of teachers or aides.)

- Write an inequality for each condition given in this situation.
- Graph the inequalities on the same coordinate axes.
- If the school board hires 12 teachers and 5 aides, will all of the conditions be satisfied?

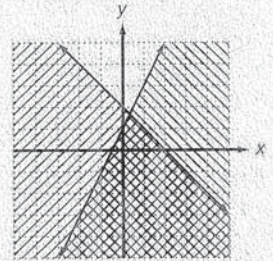
23. **Multi-Step Problem** The math club will sell rolls of wrapping paper and ribbon as a fundraiser. They can order up to 300 items, as long as they spend less than \$600. They also must buy at least as many rolls of wrapping paper as rolls of ribbon. Each roll of wrapping paper costs the club \$2. Each roll of ribbon costs the club \$3.

- Write an inequality for each condition given in this situation.
- Graph the inequalities on the same coordinate axes.
- If the club buys 150 rolls of wrapping paper and 100 rolls of ribbon, will all of the conditions be satisfied?

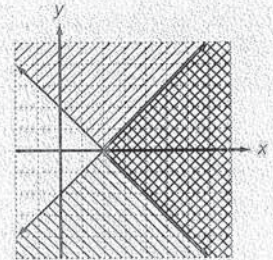
3. PRACTICE/ASSESS

LESSON QUIZ

- Solve the system by graphing
 $y \leq 2x + 1$
 $x + y \leq 2$



- Solve the system by graphing
 $x + y \geq 2$
 $x - y \geq 2$



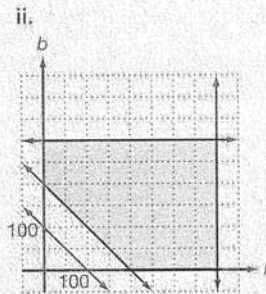
Assignment Guide

▼ Core 1–28
Extension 29–31

Use Mixed Review to maintain

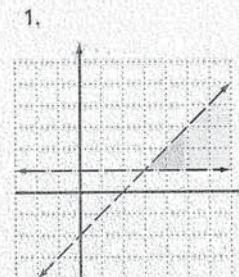
Try This

- e. i. $0 \leq b \leq 300$
 $0 \leq r \leq 400$
 $b + r \geq 100$
 $b + r \geq 200$



iii. Yes

Exercises



2–20. See Additional Answers.
Answers continue on next page.

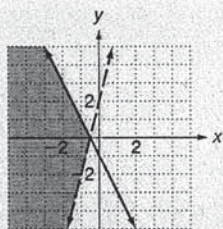
→ Pgs 658-663



Practice Multiple Choice

Choose the best answer.

1. Which system of inequalities has the shaded area as its solution?



- A $y > 4x + 2$
 $y \leq -2x - 1$
- B $y < 4x + 2$
 $y \geq -2x - 1$
- C $y \geq 4x + 2$
 $y < -2x + 1$
- D $y \leq 4x + 2$
 $y < -2x + 1$

2. Solve.

$$x > 2 \text{ and } x < -5$$

- F \emptyset
- G 3
- H {all real numbers}
- J $\{x \mid -5 < x < 2\}$

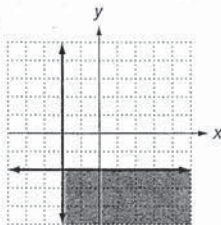
- 1. A; Algebra 9.0
- 2. F; Algebra 6.0

424 Chapter 9 Inequalities and Absolute Value

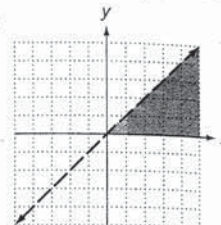
B

Write a system of inequalities whose solution is shown by each graph.

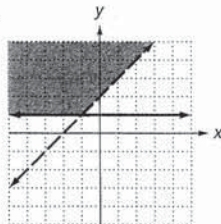
24.



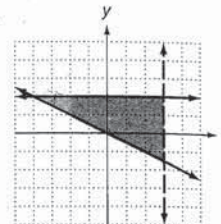
25.



26.



27.



28. **Critical Thinking** Write a system of inequalities such that the graph of its solution forms a square region with one corner at the origin and two adjacent sides on the x - and y -axis.

Challenge

The corner points of the graph of a system of inequalities are the points where the boundary lines intersect. Graph each system of inequalities and give the coordinates of each corner point.

29. $x - y \leq 1$

$$x + 2y > 1$$

$$y < 1$$

30. $x > 0$

$$2y + x \leq 6$$

$$x + 2 \leq 2y$$

31. Graph $(x - y - 3)(x + y + 2) > 0$. (*Hint: Under what conditions is a product of two numbers positive?*)

Mixed Review

Solve. 32. $x + 5 \leq 16$ or $x - 3 > 5$

33. $-7 \leq -3x + 2 < -1$

34. $|4x| > 8$

35. $|4x| < 8$

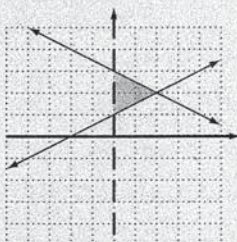
36. $|2y + 1| > 9$

37. $|2y + 1| < 9$ 9-2, 9-4

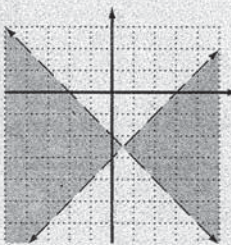
Factor. 38. $x^2 + 3yx + 2y^2$ 39. $b^2 + 4b + 4$ 40. $9x^4 - 25y^2$ 6-2, 6-4, 6-7

41. A collection of dimes and quarters is worth \$16.20. There are 81 coins in all. How many are dimes and how many are quarters? 8-6

30. (0, 1), (0, 3), (2, 2)



31.



Mixed Review

32. $x \leq 11$ or $x > 8$

33. $1 < x \leq 3$

34. $x > 2$ or $x < -2$

35. $-2 < x < 2$

36. $y > 4$ or $y < -5$

37. $-5 < y < 4$

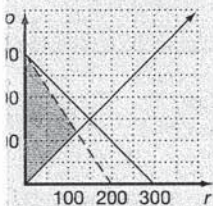
38. $(x + 2y)(x + y)$

39. $(b + 2)^2$

40. $(3x^2 - 5y)(3x^2 + 5y)$

41. 27 dimes, 54 quarters

$$\begin{aligned} + 3r &< 600 \\ + r &\leq 300 \\ &\geq r \end{aligned}$$



s, the total amount spent is not more than \$600.

$$-2, y \leq -2$$

$$), x > y$$

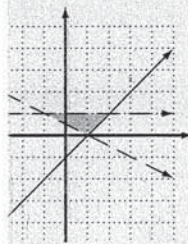
$$), y > x + 2$$

$$), y \leq 2, y \geq \frac{1}{2}x$$

ers may vary. Example:

$$; \leq 5, 0 \leq y \leq 5$$

$$1), (1, 0), (2, 1)$$



9-1

A **set** is a well-defined collection of objects called **members** or **elements**. To write a set using **roster notation**, list the elements of the set. To write a set using **set-builder notation**, specify a variable and a description of how to build the set.

Write using (a) roster notation and (b) set-builder notation.

- the set A of whole numbers less than 9
- the set B of odd whole numbers
- the set C of positive multiples of 7 less than 30
- the set D of negative integers greater than -5

The **intersection** of two sets A and B , $A \cap B$, is the set of all elements that are common to both sets. The **union** of the two sets A and B , $A \cup B$, is the set of all elements that are in A or B or both.

Let $A = \{2, 4, 8, 16, 32\}$, $B = \{4, 8, 12, 16, 20\}$, and $C = \{3, 6, 9, 12, 15, 18\}$. Find the following.

- $A \cup B$
- $A \cap B$
- Let W be the set of whole numbers and P be the set of positive odd numbers. Find $W \cap P$.
- Let Z be the set of integers and W be the set of whole numbers. Find $Z \cup W$.

9-2

A **conjunction** of two statements is formed by the word "and" connecting the two statements and is true when both statements are true. The graph of a conjunction is the **intersection** of the two individual graphs. A **disjunction** of two statements is formed by the word "or" connecting the two statements and is true when one or both of the statements are true. The graph of the disjunction is the **union** of the two individual graphs.

- Graph the conjunction $x \geq -2$ and $x \leq 1$.
- Graph the conjunction $-1 < p < 12$.
- Graph the disjunction $x < -2$ or $x > 4$.

Solve and graph.

- $-2 \leq x + 3 < 7$
- $-1 \leq x + 4 < 4$
- $12 + 2x < 0$ or $-2 - x \leq 3$
- $x + 3 < 4$ or $x - 2 > -5$

Key Terms

boundary line (p. 417)
 conjunction (p. 405)
 disjunction (p. 407)
 element (p. 400)
 empty set (p. 401)
 half-plane (p. 417)
 intersection (p. 401)
 member (p. 400)
 roster notation (p. 400)
 set (p. 400)
 set-builder notation (p. 400)
 system of inequalities (p. 421)
 truth table (p. 410)
 union (p. 402)

CHAPTER 9 WRAP UP

- $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
 - $A = \{x \mid x \text{ is a whole number and } x < 9\}$
- $B = \{1, 3, 5, \dots\}$
 - $B = \{x \mid x \text{ is a whole number and } x \text{ is an odd number}\}$
- $C = \{7, 14, 21, 28\}$
 - $C = \{x \mid x \text{ is a positive multiple of 7 and } x < 30\}$
- $D = \{-4, -3, -2, -1\}$
 - $D = \{x \mid x \text{ is a negative integer and } x > -5\}$
- $\{2, 4, 8, 12, 16, 20, 32\}$

6. $\{2, 3, 4, 6, 8, 9, 12, 15, 16, 18, 32\}$

7. $\{4, 8, 16\}$

8. $\{12\}$

9. $W \cap P = P$

10. $Z \cup W = Z$

11.



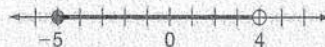
12.



13.



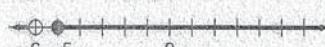
14. $-5 \leq x < 4$



15. $-5 \leq x < 0$



16. $x < -6$ or $x \geq -5$



17. All numbers on the number line



Write using (a) roster notation and (b) set-builder notation.

- the set A of whole numbers greater than 4
- the set B of negative integers greater than -3
- the set C of positive multiples of 3 less than -3

Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{5, 10, 15\}$, and $C = \{2, 3, 5, 7, 11\}$. Find the following.

- $A \cap C$
- $A \cap B$
- $B \cap C$
- $A \cup B$
- $B \cup C$

Solve and graph.

- $0 < x - 4 < 3$
- $-10 \leq x - 5 < -8$
- $x + 2 < 1$ or $x + 3 \geq 5$
- $2x < -2$ or $3x - 1 > 2$

Solve.

- $|x - 2| = 4$
- $|3x - 6| = 21$

Solve and graph.

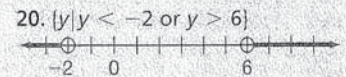
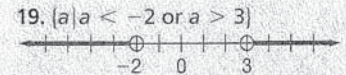
- $|3p| < 21$
- $|x + 4| < 2$
- $|9 - r| \leq 9$
- $|4x| \geq 12$
- $|2a - 1| > 5$
- $|-3y + 6| > 12$

Solve each system by graphing.

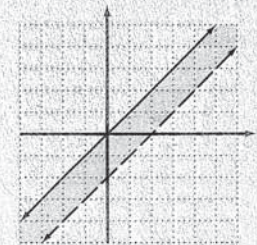
- $y \leq x$
 $y > x - 2$
- $y > -3$
 $x < -2$
- $y - x + 2 < 0$
 $2y - 2 > x$
- $x - y > 2$
 $x + y < 1$

Assessment Item Analysis

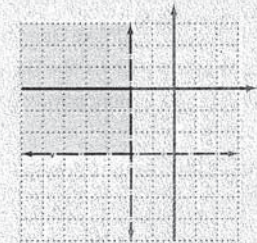
Item	Lesson
1-8	9-1
9-12	9-2
13, 14	9-3
15-20	9-4
21-24	9-5, 9-6



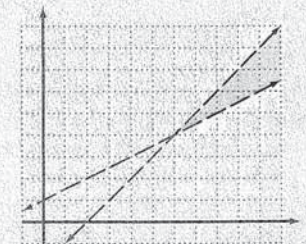
21.



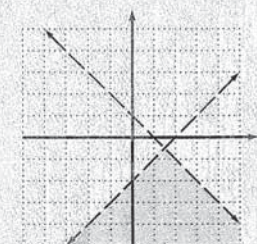
22.



23.



24.



CHAPTER 9 ASSESSMENT

- (a) $A = \{5, 6, 7, \dots\}$
(b) $A = \{x | x \text{ is a whole number and } x > 4\}$
- (a) $B = \{-2, -1\}$
(b) $B = \{x | x \text{ is a negative integer and } x > -3\}$ or $\{x | x \text{ is an integer and } -3 < x < 0\}$
- (a) $C = \emptyset$
(b) $C = \{x | x \text{ is a positive multiple of 3 and } x < -3\}$
- $\{2\}$
- $\{10\}$

- $\{5\}$
- $\{2, 4, 5, 6, 8, 10, 12, 15\}$
- $\{2, 3, 5, 7, 10, 11, 15\}$
- $\{x | 4 < x < 7\}$
- $\{x | -5 \leq x < -3\}$
- $\{x | x < -1 \text{ or } x \geq 2\}$
- $\{x | x < -1 \text{ or } x > 1\}$

- $\{6, -2\}$
- $\{9, -5\}$
- $\{p | -7 < p < 7\}$
- $\{x | -6 < x < -2\}$
- $\{r | 0 \leq r \leq 18\}$
- $\{x | x \leq -3 \text{ or } x \geq 3\}$